

Seminar: Vibrations and Structure-Borne Sound in Civil  
Engineering – Theory and Applications

**Vibrations and Structure-Borne Sound in Civil  
Engineering - Theory and Applications**

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# Contents

## Part 1.

- *The Introduction.*
- *Bottoms, classification of influences.*
- *WAVES IN BUILDING DESIGNS*
- *Methods of the analysis (calculation)*

## Part 2.

- *A modern technique of a noise isolation. Examples.*
- *STRUCTURAL COMPONENTS And UNITS*
- *SPECIAL SOUNDPROOF MATERIALS And DESIGNS*

## Part 1.

### 1. *The Introduction.*

Long-term experience of some the European countries confirms necessity of use of gobos at repair or construction of buildings. They are important the same as also heat-insulating materials with a view of improvement of acoustics and performances of suppression of noise.

Until recently in Russia not enough attention was given acoustics of premises. It has been connected not only with economy of means, but also with absence of the certain acoustic norms.

Earlier the purpose of designers was in most cases the maximal area of construction at the limited budget. Other factors remained, as a rule, without attention. Now cost of comfort of a dwelling unit and a building as a whole depends, including from a level of isolation of a building from external noise, sound insulation between dwelling units and sound insulation between rooms in a dwelling unit.

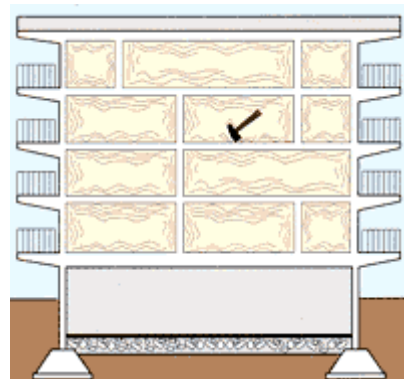
## 2. Bottoms, classification of influences.

### Ways of isolation of noise

According to radiants, noise inside of a building can be divided into some categories:

- *Air noise;*
- *Shock noise;*
- *Structural noise (sounds from building systems (system of ventilation, heating, etc.));*

**Air noise** arises at radiation of a sound (a human voice, musical instruments, machines, the equipment, etc.) in air space which reaches any protection and causes its fluctuation. Vacillating the protection, in turn, radiates a sound in an adjacent premise, and thus air noise reaches the person perceiving him.



**Shock noise** arises at mechanical influence of subjects directly on a floor slab. The given noise is caused by simple walking of people on a floor or moving on a dwelling unit of pets, movement of subjects - in common, quite natural and lawful

operations which manufacture cannot be regulated on time of day or on a level of created noise.

**Structural noise** arises at contact of building designs to the various vibrating equipment (Machine tools, extracts, audiocolumns of high capacity). Structural noise is spread on building designs and radiated in premises on all ways of the distribution.

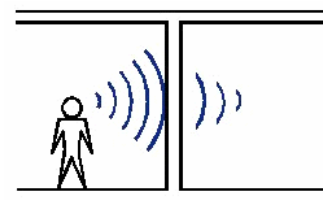


FIGURE 2. Noise Reduction between two spaces by a dividing wall.

Velocities of distribution of a sound of various materials	
Material	Velocity of a sound
Steel	5000 m/s
Concrete	3000 m/s
Firm lumber	1500 m/s
Water	1000 m/s
Air	340 m/s

Конец Слайд 4

Walls not so often treated to an operation of shock noise, unlike a floor which perceives both sound noise, and shock noise (circulation, rearrangement of furniture, etc.). In such cases energy of shock noise is much more, than energy of air noise. Therefore, unlike air noise, the problem of shock noise cannot be solved by increase in mass of designs.

Actual isolation of air noise depends not only on sound-proof properties of a design of a protection, but also from the area of this design, and also from a sound absorption of a surface of walls, a floor, a ceiling and subjects in. As exponents in each concrete case vary, is entered.

Concept of soundproofing ability (own sound insulation) R which is measured in decibels. This quantity does not depend neither on the area, nor from a sound absorption, it is inherent only in the most protecting design.

More in detail views and ways of sound insulation will be considered in the second part of the report.

### **According to views of sound waves caused by vibration**

Sound insulation of premises in civil buildings is substantially defined not only design data of the protecting designs dividing premises, but also conditions of distribution of sound vibration on adjacent designs of a building. Acoustic and vibrating radiants of a sound cause in a design of a building sound vibration in the form of elastic waves:

- Cross-section
- Longitudinal
- flexural mode.

Combination of some factors, such as the high module of elasticity and low dissipative properties of materials of walls and floorings, rigidity of their joints, presence in a building enough powerful radiants of noise and vibration, lead to infringement of conditions of acoustic comfort in premises.

## **3.WAVES IN BUILDING DESIGNS**

Distribution of elastic waves to building designs as in systems with the distributed parameters of mass and rigidity, is based on regularities *of the theory of elasticity*. Elastic properties of structural materials are practically equally shown in static and dynamic modes of loading (change of an oscillation frequency means). They are well enough studied, and consequently their values are resulted in many directories. Definition of parameters of distribution of waves is made from the analysis of the is intense-deformed state in constructive parts and record of wave equations of balance of internal and external forces.

The most spread building designs - slabs, panels and girders - in the technical mechanics are considered as isotropic plates and cores. Distribution of waves to them has a complicated picture.

Combinations of longitudinal and cross-section waves allow to consider the some idealized types of waves in building designs:

- shear modes
- quasi-longitudinal
- flexural.

**Shear modes waves** are accompanied by fluctuation of particles to perpendicularly direction of distribution of a wave. Interest represents distribution of shift waves along a principal axis of a core or along one of longitudinal axes of a plate.

**Quasi-longitudinal** waves are accompanied not only longitudinal displacement in a direction of distribution of the wave, conterminous with one of longitudinal axes of a design, but also cross-section strains of a structural section. Influence of cross-section strains essentially increases on high frequencies at the significant thickness of a core or a plate.

Distribution of **flexural waves** causes turn of sections, and also cross-section displacement of section is perpendicular to a direction of distribution of a wave and a longitudinal axis of a design. Phase velocity flexural waves as, **however**, and longitudinal depends on an oscillation frequency, and character of this dependence named dispersive, differently is shown on low and high frequencies.

The building panels executed as a single-layered flat design without edges of rigidity, are considered as isotropic plates with height of section  $h$ . As well as for girders, for plates it is possible to allocate two frequency ranges: the low frequencies supposing a series of simplifications of the elementary theory, and high frequencies on which it is necessary to consider a common picture of a field of waves or to represent it in the form of a combination of waves idealized types. The border of ranges is displaced in area of low frequencies with increase in thickness of a plate.

#### **4. Methods of the analysis (calculation)**

The methods of calculations of fluctuations most spread in a modern practice and sound generations of complicated engineering designs are *methods of final and boundary elements (MFE, MBE)* and a power method (PM) (A power method often name a statistical power method (SPM), meaning features of assumptions at its application). These methods are realized in the form of commercial programs: ANSYS, SYSNOISE, ABACUS (MFE), AutoSEA, SEAM, SEADS (PM), etc.

MFE basically is exact enough and universal method as in its bottom the fundamental equations of the theory of elasticity lay. But at its use there are problems computing nature-when quantity of the elements necessary for the correct description of a design, becomes very much

greater (especially for enough complicated designs with increase of frequency). *Therefore in practice MKЭ it is applied usually on low frequencies, and ЭМ - on average and high.*

At calculation PM the stationary power state of mechanical system is considered. Generated balance of energies, entered and absorbed in system, and also passing from one subsystem in another. Thus are the used energies, average on time, space of subsystems (length, the area, volume) and to frequencies (prosummarized in frequency bands). The basic advantages of a method are its stability to an inaccuracy of input datas and absence of necessity for the detailed description of system. Therefore it is attractive to engineering calculation of a sound and vibration in complicated designs.

However ЭМ - essentially approached method. Besides confidants Coefficients of power communication applied in a method are between Subsystems. They are gained analytically, as a result of a solution of the certain problems at which statement those or other assumptions are used. Use of such coefficients is limited by a frequency band in which assumptions are fair. Therefore it is considered, that use ЭМ, in particular СЭМ, it is limited by the certain frequency band, though the conservation law of energy - bottom ЭМ - is fair on any frequencies. Other deficiency of application of analytical solutions for definition of coefficients of communication consists that frequently they do not consider features of real designs which can essentially differ from the academic models. Application to such designs of known analytical solutions, obviously, gives or rather approached estimations, or cannot be proved at all. Specified deficiencies ЭМ are not an obstacle for use of a method on To expert. As, first, engineering calculations basically confidants, secondly, are available ways for increase of precision of calculations.

*One of perspective ways of perfection ЭМ is use MKЭ for definition of coefficients of power communication.* For modelling the separate connected subsystems it is required much less finite elements, than for modelling complicated system as a whole. Therefore solution MKЭ is possible with use of a "ordinary" computer down to enough high frequencies where to system as a whole it is applied ЭМ.

The grouping of waves in a design is necessary for the further calculation. These calculations are presented by my colleagues. Later, if there will be time I shall stop on this theme more in detail.

## **Part 2.**

### **1. A modern technique of a noise isolation. Examples.**

#### **1.1. Solutions of problems of various noise influences. Sound insulation and its views**

**Sound insulation of a building - set of actions on decrease in the noise level getting into premises from the outside. The quantitative measure of sound insulation of the protecting designs, expressed in decibels, refers to as soundproofing ability.**

First of all it is necessary to notice that application of various methods of sound insulation directly depends on length, and consequently also frequencies of a wave.

All frequency range of sound waves can be divided into three parts:

- infrasonic waves (frequency up to 20 Hz). To such frequencies of waves there correspond enough greater lengths. So greater, that the basic soundproofing solutions appear unsuitable. The wave simply bends around them. In this case sound insulation is influenced only with increase in thickness and the area of the design. The given type of waves causes negative reaction of the person, as a rule is not perceived by ears. Causes feeling of anxiety. At the long influence chronic illnesses. The given range as a rule proves near to factories, large construction sites, etc.
- heard range (from 20 up to 20000 Hz). The Wave band perceived by a human ear. Here are applicable the basic receptions of sound insulation.
- ltrasonic waves (over 20000 Hz). As a rule, it is isolated in set with heard waves.

Sound insulation of premises in buildings depends not only on soundproofing ability of separate designs, but also from conditions of distribution of sound vibration on designs.

First of all it is *the functional organization* of a building providing corresponding separation or overlapping of processes, connected with noisy or silent conditions (noise from sanitary-engineering and plumbing system). The key rule providing acoustic comfort, the grouping of silent and noisy premises in corresponding functional zones and separation of these zones by the premises which are carrying out buffer function is.

. *Constructive solutions* are the second factor influencing sound insulation of premises. Sound insulation substantially depends from sound conductivity a constructive skeleton of a building. In turn, *sound conductivity* designs of buildings depends on their homogeneity. The greatest звукопроводностью one-piece buildings possess. Smaller conductance of a sound.

brick buildings with massive wall designs possess.

Effect can give:

**Vibrodamping masses;**

**The significant difference in thickness and the superficial density of the interfaced designs;**

**Use of sound-proof linings in joints of designs;**

**Application of coverings on ledge on walls and ceilings, and also designs of floating floors**

**Special porous facing materials.**

## 2.STRUCTURAL COMPONENTS And UNITS

Constructive systems of civil buildings are characterized by the scheme of distribution between elements of bearing and protecting functions, type of structural components, technological indications. Character of constructive system defines a degree of acoustic interrelation of structural components and, **as a result**, a view of mathematical model of a building under the theory of the statistical power analysis.

The basic structural materials of buildings are concrete, a brickwork, metal, a tree. Physicomechanical properties of the basic structural materials practically do not depend on

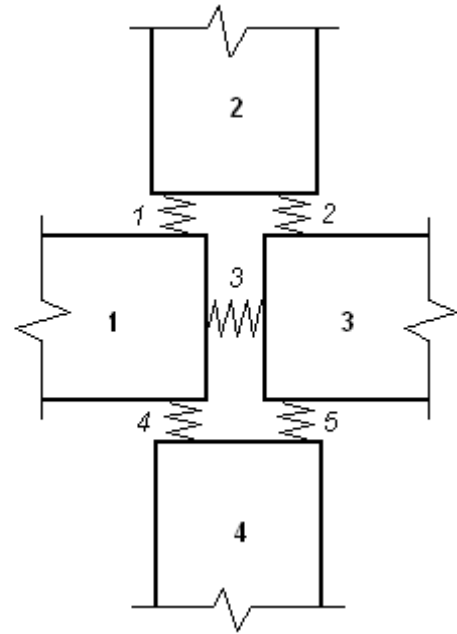
frequency of loading, therefore the dynamic module of elasticity differs from the module of elasticity measured at static loading a little.

Constructive units of buildings can be classified on

- To type of designs,
- Their forming, and
- To character of filling of joints.

*Designs* of a building can be divided into *types*: rod (columns, crossbars), plates (panels, slabs) and environments. Constructive units can be rigid or pliable. Rigid joints have filling with the same material, as a material of designs and transfer all views of movings to adjacent designs. Rigid joints are characteristic for one-piece buildings and buildings with a welded metal framing.

*Fig. 1. The universal rated scheme of a joint*



For unification of rated models of joints of a building it is expedient to select the universal shape of joints both for rod elements, and for panels. The joint having the most common scheme, should have, **apparently**, such system of interrelations when each of interfaced panels is connected with next through an elastic element. On fig. 1 the scheme of a joint of four panels connected among themselves through five elastic elements is presented.

Representing rigidity of communications between separate elements equal to zero, it is possible to exclude separate elements from a joint and thus to gain T-shaped, angular or linear joints. For joints with elastic inserts their rigidity is characterized by the module of elasticity of a material of an insert and its geometrical performances. Equating the module of elasticity of a material of an insert to the module of elasticity of a material of the basic designs, it is possible to gain a rigid joint.

Alongside with structural, insulation materials, including heat-, gobos and hermetics are widely used also. In difference from structural materials, they are characterized by low value of the dynamic module of elasticity

### 3.SPECIAL SOUNDPROOF MATERIALS And DESIGNS

Essentially scopes of soundproof materials can be grouped by several criteria.

- 1) .Concerning devices soundproofing конструкции-as already it has been told earlier (виброзадерживающие masses; the significant difference in thickness and the superficial density of the designs interfaced in a joint; use of sound-proof linings in joints of designs;



application of coverings on *относе* on walls and ceilings, and also designs of floating floors).

- 2) .Concerning spheres of application (isolation from external walls, internal parting walls, *перекрытий*-ceilings and floors, windows and doors, etc.)
- 3). Concerning the shape of release (rolled *напр. мембраны, маты*, linings, boards, slabs, etc.)

As the same designs can be applied in various structural parts of a building - it is necessary to begin, grouping materials and solutions on more common principle-to sphere of their application.

### **3.1 Sound insulation of internal parting walls.**

Inside of a building sound-proof materials are applied basically in designs of sound-proof facings internal surfaces of premises and the technical devices demanding decrease of a level of noise (installation of ventilation and an air handling, etc.). Also for improvement of acoustic properties of premises (auditoria, audiences and so forth). As a rule to such materials requirements in absence of toxicity, ecological compatibility (application of natural materials), to aesthetically comprehensible performance (are applied at facing), simplicity of installation (from for opportunities of repair), fire safety, to the sizes, etc

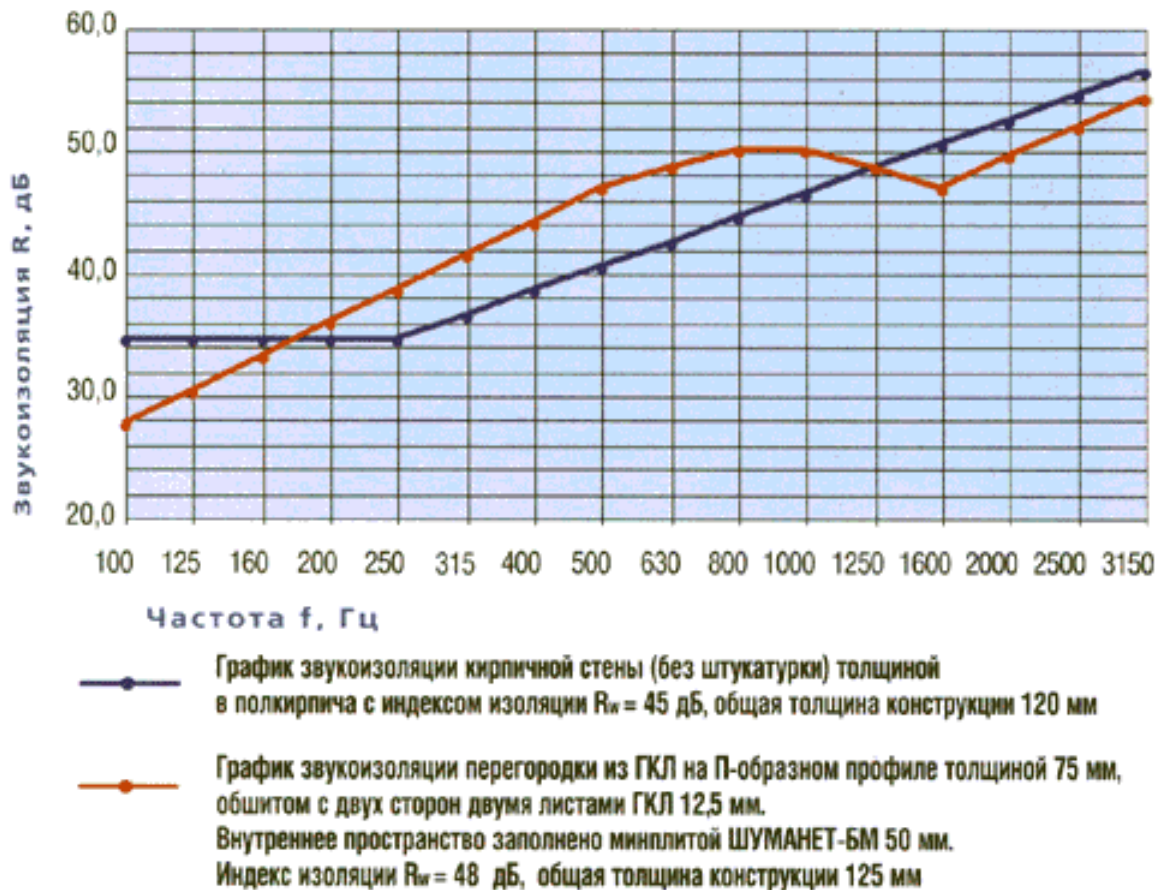
From the point of view of a structure sound absorbers can be divided into following groups:

- Porous (including fibrous);
- Porous with the punched screens;
- Resonant;
- Layered designs;
- Piece or volumetric.

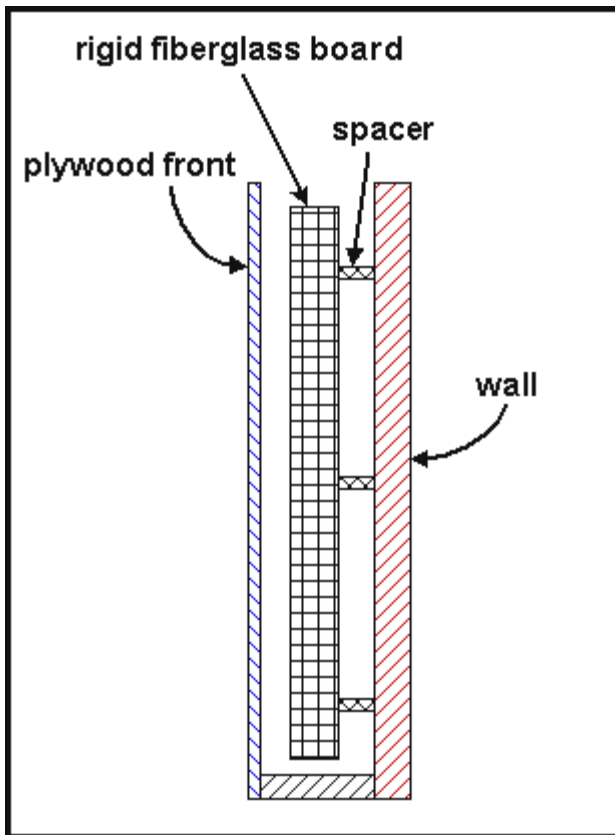
The most spread because of ease in installation porous sound absorbers make in the form of slabs of easy and porous mineral piece materials - pumices, *вермикулита*, каолина, slags, etc. with cement or others knitting which fasten to vertical or horizontal surfaces is direct or on a ledge. Such materials are strong enough and the foyer, stair flights of buildings can be used for decrease in noise in corridors.

Efficiency of sound-proof materials is estimated by coefficient of a sound absorption **a**, equal to the attitude of quantity of the absorbed energy to total of energy of sound waves falling on a material.

**The effective *principle of sandwich constructions* is often applied to sound insulation of internal walls and parting walls.**



In connection with requirements of ecological compatibility and fire safety on the foreground here there is an application of nonflammable fibrous materials. The Typical easy parting wall with good acoustic properties is a *sandwich-panel* in which fibrous sound insulator is concluded between two гипсокартонными sheets (or other sheet material). As a filler use slabs from fiber glass of firms ISOVER and PFLEIDERER, from mineral cotton wool ROCKWOOL and PAROC, and also acoustic materials with layered or cellular structure of other firms. The parting walls included in a composition these materials are capable to improve its soundproofing ability essentially. More complicated designs of parting walls, for example, with repeated alternation of layers of gypsum cardboard and sound insulator or even an air layer between two layers of a sound-proof material are possible also. Sandwich constructions of parting walls are more dear and are complicated in installation, but provide the greatest possible sound insulation. For maintenance of good sound insulation between premises of a parting wall it is impossible prop up on pure floors or logs, and it is necessary to establish directly on a floor slab. And, to bring to nothing probability of occurrence of resonant fluctuations, a parting wall do not lead up to a ceiling on 15-20 mm, filling the remained backlash an elastic gobo. The same pillow should be stipulated and on a line of a support of a parting wall on a floor.



**Figure 7:** Sound striking the plywood front panel causes it to vibrate. The fiberglass then damps that vibration.

### **3.2 Sound insulation of external walls**

In itself external load-carrying structures due to the massiveness are good звуко- and вибро-insulators. Homogeneous walls (panel houses) spend a sound and it is better than vibration than combined teams (a brickwork, various combinations with facing-sandwich-panel).

Unlike internal parting walls, to sound insulation of external load-carrying structures other requirements are applied.

From additional it is possible to allocate stability to penetration of an external moisture, but thus good vapor permeability (for that that internal pairs would not be condensed inside of a wall and did not lead to its destruction). As heat-insulating properties are important.

Requirements to naturalness, design aesthetics, simplicity of installation, etc. play here a smaller role.

The principle described above *sandwich* designs here is primary, in communication by the basic role of external walls in thermal protection. It is possible to tell that its application inside of premises is borrowed, at complex warmly-sound insulation all building.

Other principles remain the same, certainly it is impossible to suppose the slightest cracks and apertures. (as a rule with them struggle спец. Fillers).

The special attention should be given areas of windows. (about them below).

### 1.3. Sound insulation of floorings

The noise isolation of a premise from below and from above is defined by interfloor flooring. However for protection against structural noise it should be made too thick and heavy. As additional sound insulator it is possible to mount a pendant ceiling. And here between a bottom slab and a floor covering (a parquet, a lino, a laminate etc) usually mount a design of a "floating" floor. It will noticeably reduce noise of steps, falling of subjects, etc. shock noise.

#### 1). Increase in isolation of air and shock noise by flooring from the below located premise

a). When two vertically located premises (dwelling unit) belong to different owners, to agree about joint performance isolation of noise works frequently it is impossible. And as the neighbour always suffers from shock noise from below, it should pay off for someone's carelessness. Thus it is important to represent, that 20 дБ those decrease in a level of shock noise which could be easily gained at the device of a floating floor from a upper floor, cannot be made up by any means from a ground floor. Practice shows, that efficiency actions of noise isolation "from below" seldom exceeds quantity 15 дБ.

b). In case of an arrangement on ground floors of residential buildings of noisy premises - the cafe, restaurants or shops - arises a return problem. It is required to protect above located inhabited a premise from loud sounds (air noise), especially at night. Also at the device of a sound department or any other premise where the full silence should be provided, interfloor flooring should be bindingly in addition isolated. And for noise of any type.

*For today of one of the most effective designs of an additional noise isolation it is considered a pendant ceiling from fibre reinforced gypsum panels (FRGP) with articulate bracketes in a combination to the pendant sound-proof ceiling located below.*

**The mentioned below scheme of an additional noise isolation of flooring was applied in one of the Moscow restaurants, located on a ground floor of a residential building.**

The measured index of isolation of air noise by an initial design of the flooring representing multihollow reinforced concrete slabs by thickness of 220 mm, has made  $R_w = 48$  дБ (rated value  $R_w = 52$  дБ). According to operating СНиП the index of isolation of air noise by flooring between inhabited dwelling units and the restaurants located below should be not less  $R_w = 62$  дБ. Thus, by means of a design of false ceilings it is necessary to add not less  $R_w = 14$  дБ.

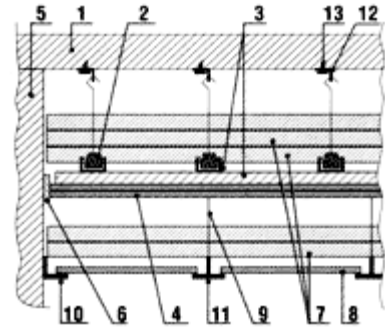
The following design of pendant ceilings (fig. 2) has been offered:

- To a floor slab (poses. 1) on adjustable brackets "ТИГИ-Кнауф" (poses. 2) it is suspended потолочный a structure of software--60/27 (poses. 3). The brackets step is equal 500 mm. Thus all brackets (poses. 2) have articulate bracket through corners with apertures (poses. 12) which one party fasten to a floor slab on anchor dowels (poses. 13);
- To a structure of ceiling strengthen two layers **fibre reinforced gypsum panels** sheets (poses fasten. 4) thickness on 10 mm. Thus of a place of joining of sheets **FRGP** with walls (poses. 5) of 6 mm (poses are carried out without parietal directing structures through vibration-isolating a lining "Vibrosil-K" thickness. 6)(poses. 6);
- width the given design of a false ceiling from ГВЛ from a floor slab makes about 400 mm. 3 layers of sound-proof cotton wool keep within space between a floor slab and sheets ГВЛ "Шуманет-БМ" thickness on 50 mm (poses. 7);
- Below a false ceiling from ГВЛ on width 400 mm are suspended a sound-proof false ceiling of mark "Ecophon" model "Harmony" (poses. 8) on own pendant system (poses. 9, 11) with application a parietel corner (poses. 10);

- Atop of slabs of a false ceiling "Ecophon" stack two layers of sound-proof cotton wool "Шуманет-БМ" thickness on 50 mm (poses. 7).

Fig. 2 Design of an additional noise isolation of flooring

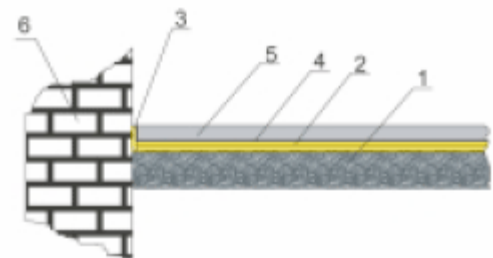
1. A floor slab;
2. Adjustable bracket "ТИГИ-Кнауф";
3. overhead a metal structure PP 60/27;
4. layers **fibre reinforced gypsum panels** sheets thickness of 10 mm;
5. A wall;
6. vibration-isolating a lining "Vibrosil K" thickness of 6 mm;
7. Sound-proof cotton wool "Шуманет БМ" thickness of 50 mm;
8. A tile of a sound-proof ceiling "Ecophon Harmony" thickness of 20 mm;
9. bracket designs of a ceiling "Ecophon";
10. parietal a corner "Ecophon";
11. The main structure of a design of a ceiling "Ecophon";
12. A corner with apertures for a bracket;(pos.2 goes hier but is not fixed!!!)
13. An anchor-dowel.



As for maintenance of necessary sound insulation quality of civil and erection works is rather important; even the most inappreciable cracks, apertures, cracks in designs sharply worsen sound-proof properties of the last

## 2). Increase in isolation of air and shock noise by flooring from the **Above** located premise

Now the most effective means of struggle against shock noise is application of a design **of a "floating" floor**. Designs concern to this type of floorings with a continuous elastic layer between a floor both a bearing reinforced concrete slab and designs with a floor on soft and elastic linings. Concerning materials of a floating floor it is necessary to carry the raised strength and elasticity to the basic requirements, in connection with loadings non-comparable concerning walls.



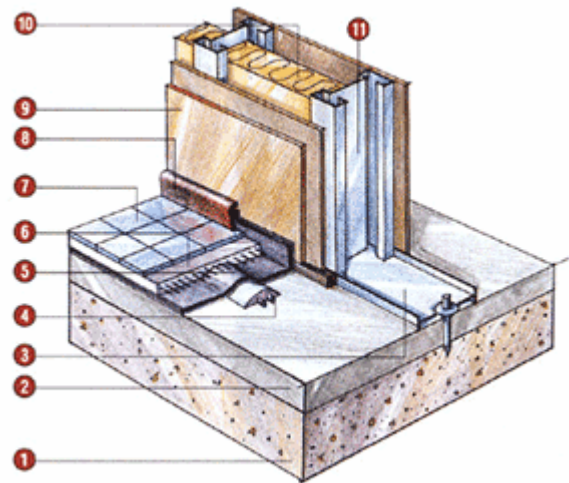
The soundproofing floor can be mounted on logs or on the elastic ("floating") warrant. Shock noise reduce by means of a substrate from various materials. For example, from полимерно-bitumen membranes Fonostop Duo (firm INDEX), a technical fuse thickness up to 8 mm from firm ИРОСОРС or sheets "Регупол", executed from a rubber crumb and полиуретана ("РЕГУПЕКС"). From above do a concrete coupler by thickness of 30-50 mm, and already on it lay a fair floor covering. Due to the small module of elasticity of a material of a substrate distribution of shock noise sharply falls.

ТИГИ-KNAUF Offers the isolating noise "pie". Various combinations of its layers in a combination to a leaf of polystyrene thickness of 20-30 mm allow to change index  $L_{nw}$  on 20-30 дБ for vibrations with frequency of 150-3000 Hz. On the average "floating" floor is capable to reduce this index on 8-33 дБ for the noise most spread in a life with frequencies from 150 up to 3000 Hz.

In Addition 4 is considered the principle of installation of a floating floor.

The scheme of a combination of two most typical soundproofing designs: a multilayered parting wall and a "floating" floor

1. A floor slab
2. A leveling coupler
3. Metal directing
4. A warm floor
5. noise -and a hydroisolating lining
6. A coupler
7. A tile
8. A plinth
9. Gypsum cardboard
10. A sound-proof filler
11. Metal racks with step of 600 mm.



## References

1. Zavarikin V. M., Gitimirsky V. G., Lapchik M. P. Numerical methods. — M.: Prosveschenie, 1990.
2. Kovrigin S.D., Krishkov S. I. Architectural acoustics-building. — M.: Visshaia shkola1986.
3. Ovsianikov Architectural acoustics-building S. N. Distribution of sound vibration to civil buildings. — Tomsk: TGASU, 2000. — 378 c.
4. MSDN Library – January 2002.
5. Room acoustic modeling written by Philip Edelbrock. — <http://www.ece.orst.edu/~edelbrp>
6. C. Hopkins. Statistical energy analysis of coupled plate systems with low modal density and low modal overlap. JSV. 2002, V. 251, No 2, pp. 193–214.

7. C. Simmons. Structure-borne sound transmission through plate junctions and estimates of SEA coupling loss factors using the finite element method. JSV. 1991, V. 144, pp. 215–227.
8. L. Cremer, M. Heckl, E. E. Ungar. Structure-Borne Sound, 2nd edition, Springer, Berlin, 1988.
9. Liapunov V.T., Nikiforov A. S. Isolation of vibrations in ship designs. L "Shipbuilding" , 1975.
- 10 L. Cremer, M. Heckl, E. E. Ungar. Structure-Borne Sound, 2nd edition, Springer, Berlin, 1988.
11. Nikiforov A. S. Acoustic designing of ship designs, L, Shipbuilding, 1990
12. C. Hopkins. Statistical energy analysis of coupled plate systems with low modal density and low modal overlap. JSV. 2002, V. 251, No 2, pp. 193–214.
13. C. Simmons. Structure-borne sound transmission through plate junctions and estimates of SEA coupling loss factors using the finite element method. JSV. 1991, V. 144, pp. 215–227.
14. L. Cremer, M. Heckl, E. E. Ungar. Structure-Borne Sound, 2nd edition, Springer, Berlin, 1988.
15. I. V. Grushezky, A. V. Smolnikov Comparison of a method of finite elements and statistical power method for calculation of vibration in periodic designs. ЦНИИ им. акад. А.Н. Крилова