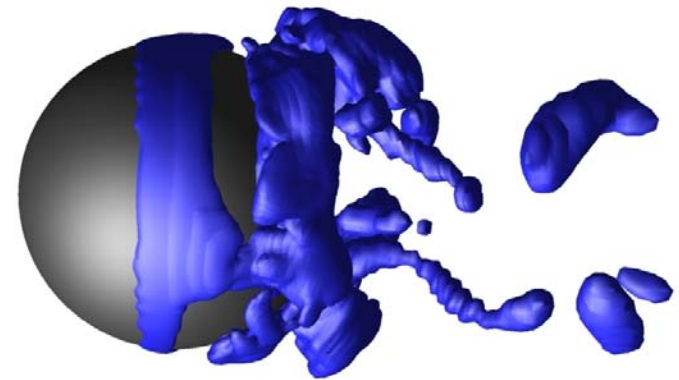
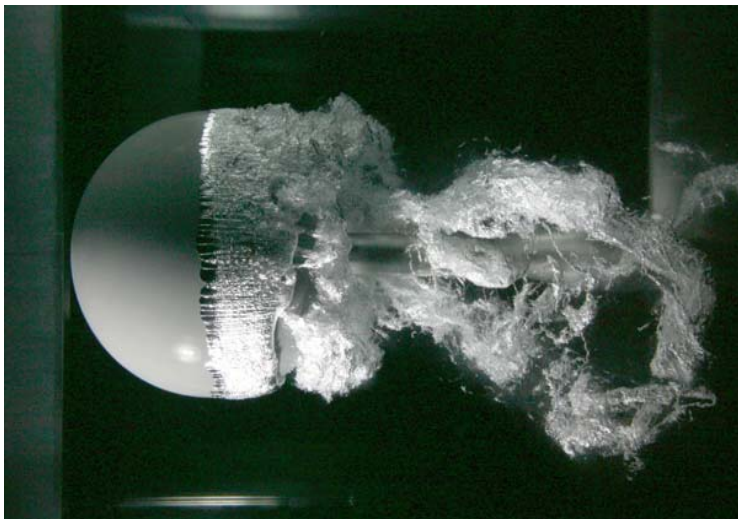


# Modelling and Computation of Dynamic Phase Transition of Liquids - Compressible Flows with Cavitation -



Michael Mihatsch  
JASS 2009

09.03.-19.03.2008, St. Petersburg, Russia

## Outline

A thick red arrow pointing to the right, highlighting the 'Introduction' section.

### Introduction

Important numbers

Physical effects

### Modeling

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### Numerical results and validation

- Spherical body:

- Hydrofoil

- Prismatic body – cavitation erosion

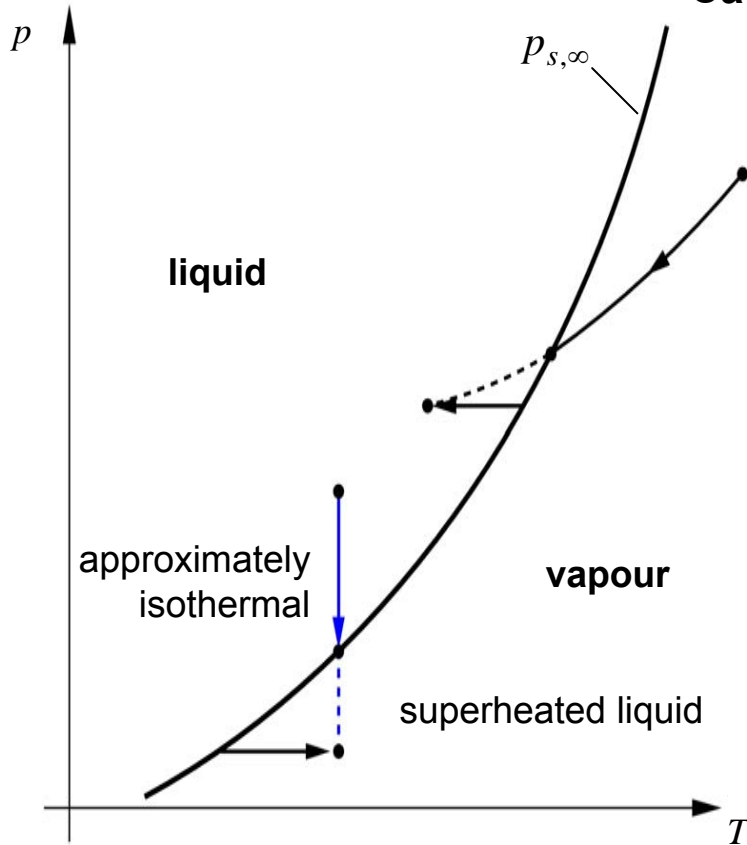
## Physics of **cavitating** flows

initially single-phase liquid fluid	$p_l \approx O(1-10^3 \text{ bar}), T \approx 300-400 \text{ K}$
	$p_v \approx O(10^{-2}-1 \text{ bar})$
	$\rho_v \approx O(10^{-2}-1 \text{ kg/m}^3)$
	$\rho_l \approx O(10^3 \text{ kg/m}^3)$
void fraction	$0 \leq \alpha \leq 1$
speed of sound	$c \approx O(1-10^3 \text{ m/s})$
strong variation of the Mach number	$M \approx O(0-10^1)$

### Dominating

- strong density variation  $\rho_l/\rho_v \approx 10^4$
- strong variation of speed of sound  $c_l/c_{\min} \approx 10^3$
- coexistence of compressible and weak compressible flow regimes
- formation of violent shocks in collapse region
- intense noise, vibration and erosion

## Cavitation dynamics



### 1. Process

Depressurization - evaporation  
 increase of volume - 1 : 50000  
 displacement of liquid fluid  
 instability

### 2. Process

collapse  
 implosion of bubbles and cavitation patterns  
 violent shocks  
 erosion

### Compressibility

local very low wave speed  $c \leq 10$  m/s  
 stiffness because of coupling  
 with regimes of  $c \approx 1500$  m/s

$P_{s,\infty}$  coexistence of phases  
**equilibrium**

----- metastable  
**time scale of flow**

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## Nucleation

### Homogeneous

Nucleus consists of molecules

Exclusive hom. nucleation allows high surface tension and highly metastable states

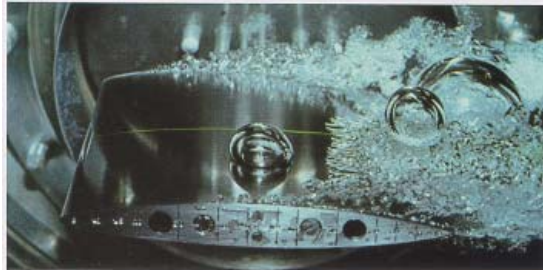
Only important with pure water

### Heterogeneous

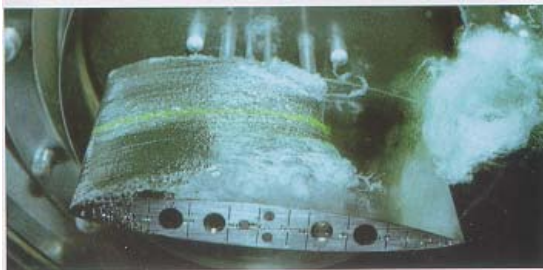
Impurities like dissolved gas or crevices at walls or particles act as nucleus

Dominant in most technical applications

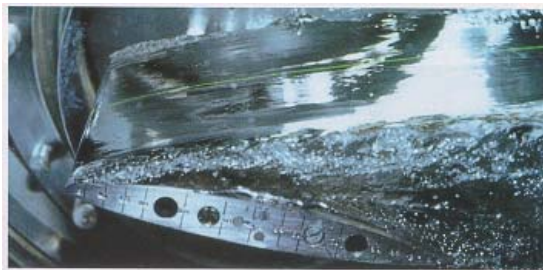
## Cavitation phenomena



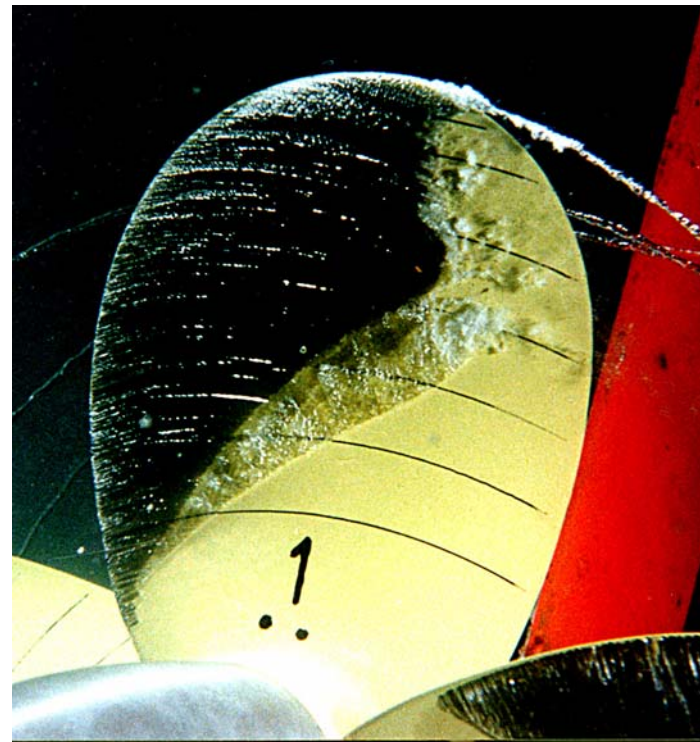
Bubble and cloud cavitation



Sheet and cloud cavitation



Supercavitation



Vortex  
 cavitation

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**Numerical results and validation**

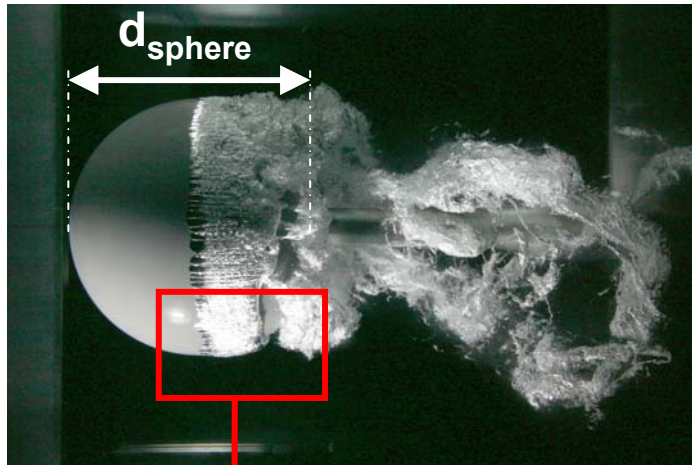
- Spherical body

- Hydrofoil

- Single bubble collapse

- Prismatic body – cavitation erosion

## Size of cavitation structures

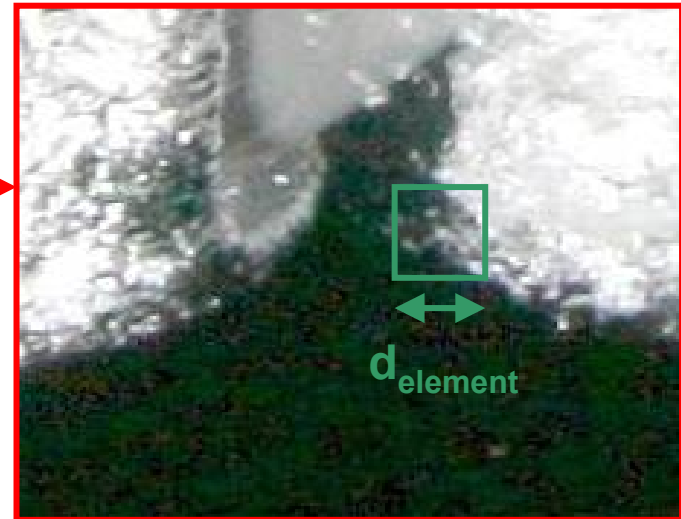
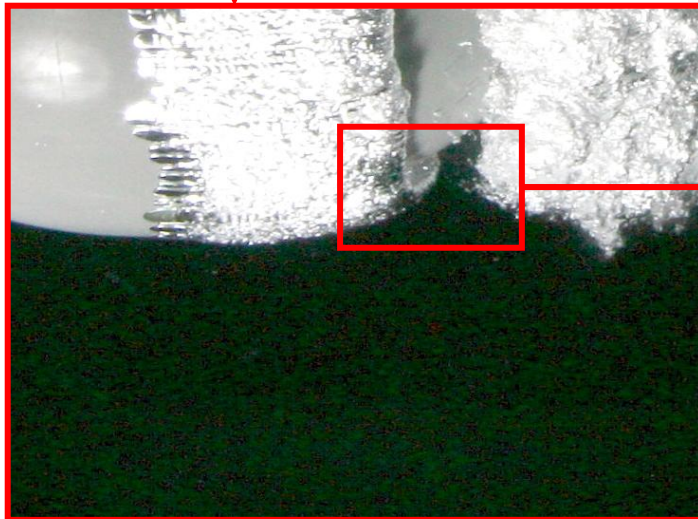


Sphere diameter

$$d_{\text{sphere}} = 1,5 \cdot 10^{-1} \text{ m}$$

Size of a fluid element

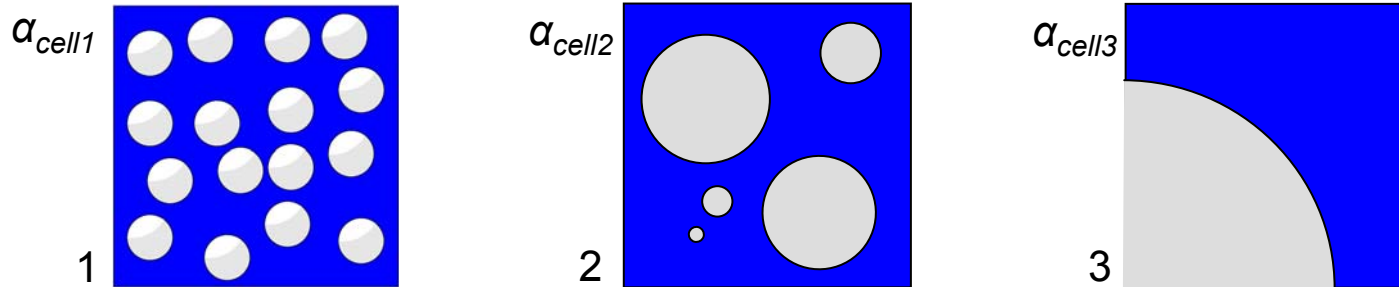
$$d_{\text{element}} = 5,2 \cdot 10^{-3} \text{ m}$$





## Two-phase flow properties via integral averages per cell

$$\text{Vapor volume fraction per cell } \alpha_{cell} = \frac{V_{vapor, cell}}{V_{cell}}$$



$$\alpha_{cell1} = \alpha_{cell2} = \alpha_{cell3}$$

subgrid scale structures → integral average properties (FVM)

$$\bar{\rho} = \frac{1}{V_{cell}} \int_{V_{cell}} \rho \cdot dV$$

$$\bar{\rho u} = \frac{1}{V_{cell}} \int_{V_{cell}} \rho u \cdot dV$$

$$\bar{\rho E} = \frac{1}{V_{cell}} \int_{V_{cell}} \rho E \cdot dV$$

Stable thermodynamic conditions → constitutive relations (EOS) determine cell variables  $\bar{p}$ ,  $\bar{T}$

$$\rightarrow \bar{\rho} = \bar{\rho}(\bar{p}, \bar{T}) \quad \bar{e} = \bar{e}(\bar{p}, \bar{T})$$

## Thermodynamic Equilibrium Conditions - Substitute EOS

- “Equation of state” for liquid water: **modified Tait “EOS”** (thermal and caloric EOS for **pure liquids**)

$$\bar{p}(\bar{\rho}, \bar{T}) = (B + p_{sat}(\bar{T})) \cdot \left( \frac{\bar{\rho}}{\rho_{l,sat}(\bar{T})} \right)^n - B$$

$$e_l(\bar{T}) = c_{vl} \cdot (\bar{T} - T_{ref}) + e_{l,ref}$$

- EOS of pure water vapour: **perfect gas law** (thermal and caloric description of **pure vapour**)

$$\bar{p}(\bar{\rho}, \bar{T}) = \bar{\rho} \cdot R_v \cdot \bar{T}$$

$$e_v(\bar{T}) = c_{vv} \cdot (\bar{T} - T_{ref}) + e_{v,ref} + l_v$$

For water:  $B \approx 3.3 \cdot 10^8 \text{ Pa}$ ,  $n \approx 7.15$ , reference state *ref.*: expected mean temperature (293.15 K).

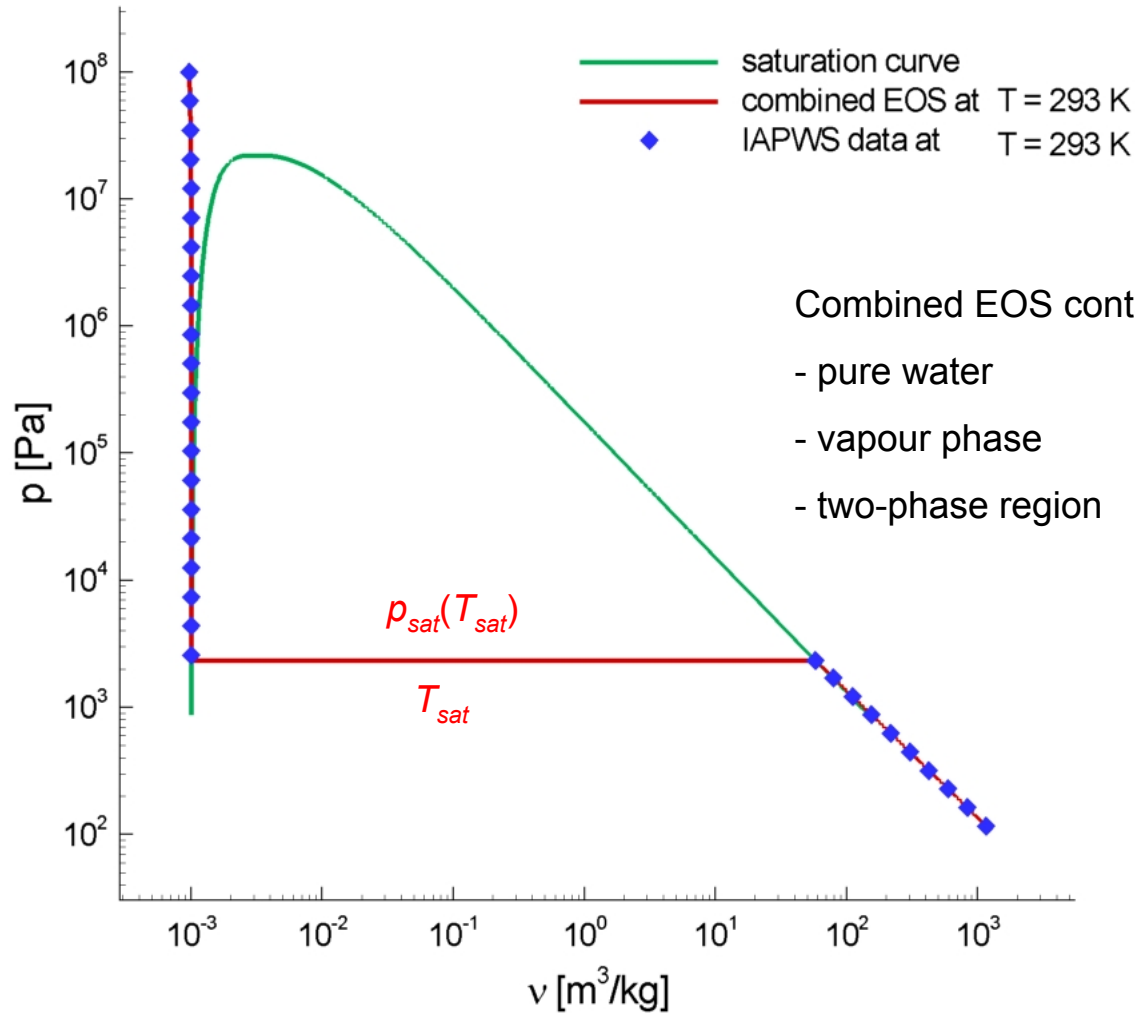
- EOS for saturated water/vapour: **saturation conditions – Oldenbourg polynomials**  
 (conditions for saturated mixture of water and water vapour for a **void fraction  $\alpha$** )

$$\bar{p} = p_{sat}(\bar{T})$$

$$\bar{\rho} = \alpha \cdot \rho_{v,sat}(\bar{T}) + (1 - \alpha) \cdot \rho_{l,sat}(\bar{T})$$

$$\bar{\rho} e = \alpha \cdot \rho_{v,sat}(\bar{T}) \cdot e_v(\bar{T}) + (1 - \alpha) \cdot \rho_{l,sat}(\bar{T}) \cdot e_l(\bar{T}).$$

## Thermodynamic model - EOS



Combined EOS contains relations for:

- pure water → modified Tait equation
- vapour phase → ideal gas law
- two-phase region → saturation conditions

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Finite volume Method

Compressible, frictionless, unsteady flows -> Euler equations

Grid: structured hexagonal cells

Flux function: density based

Solver: mod. Riemann solver

2nd order accurate

explicit

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- Spherical body

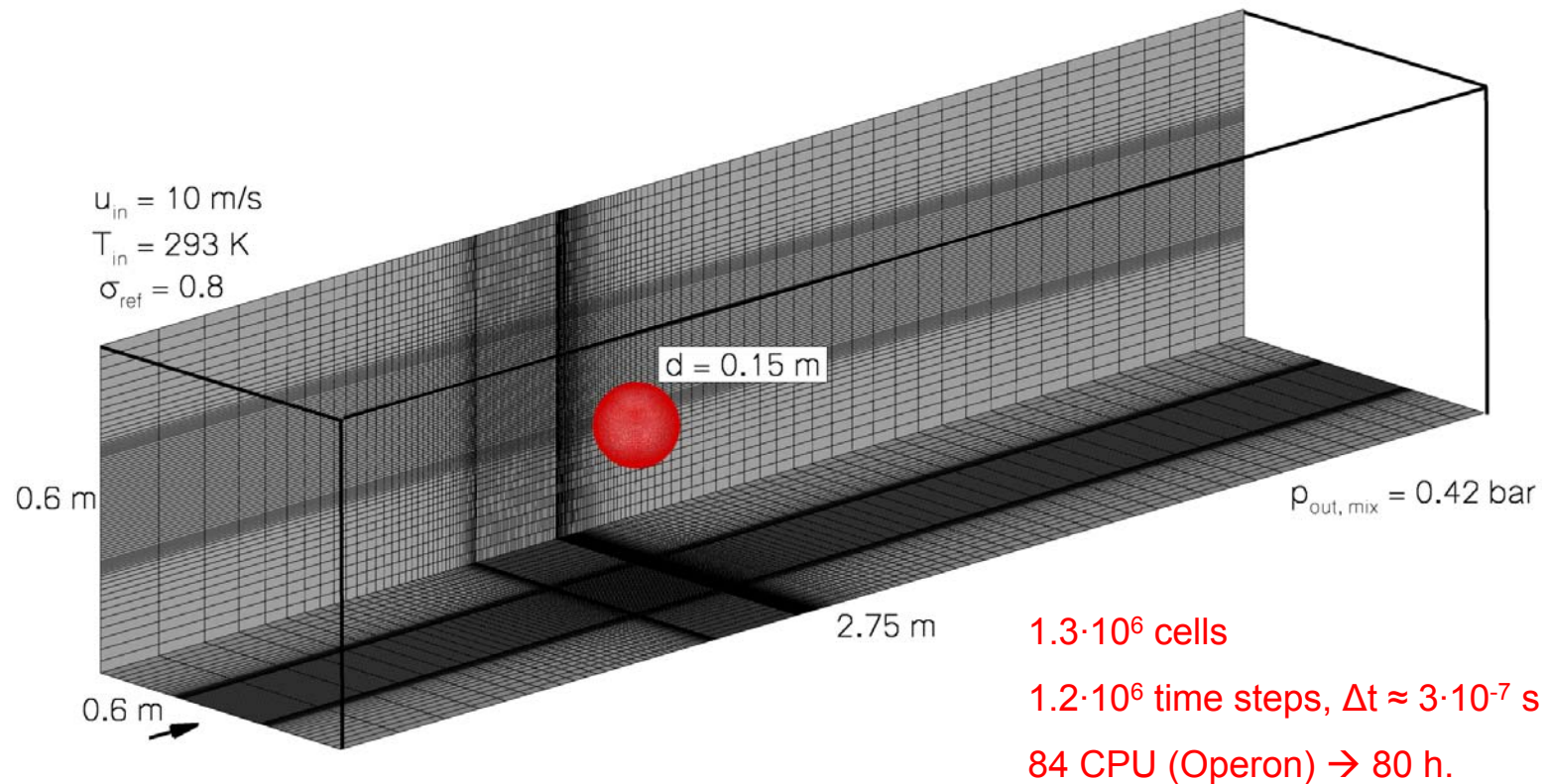
- Hydrofoil

- Single bubble collapse

- Prismatic body – cavitation erosion

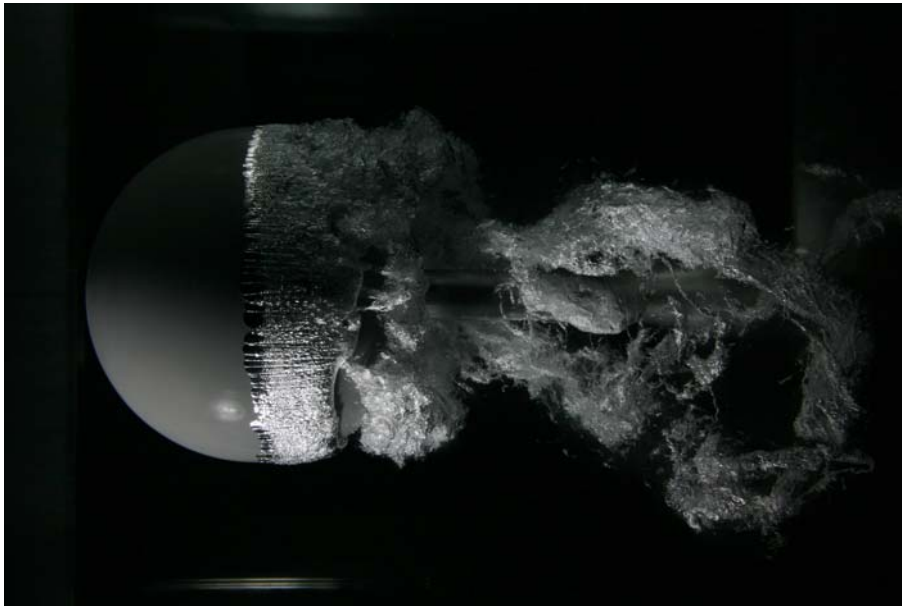
## Numerical results – application: two-phase flow

3-D simulation of Branders experiment of **cavitation around a sphere**

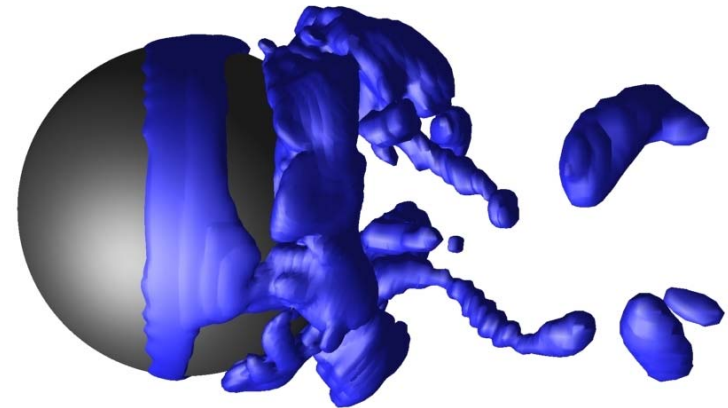


## Numerical results – application: two-phase flow

Comparison of two-phase structures experiment/simulation

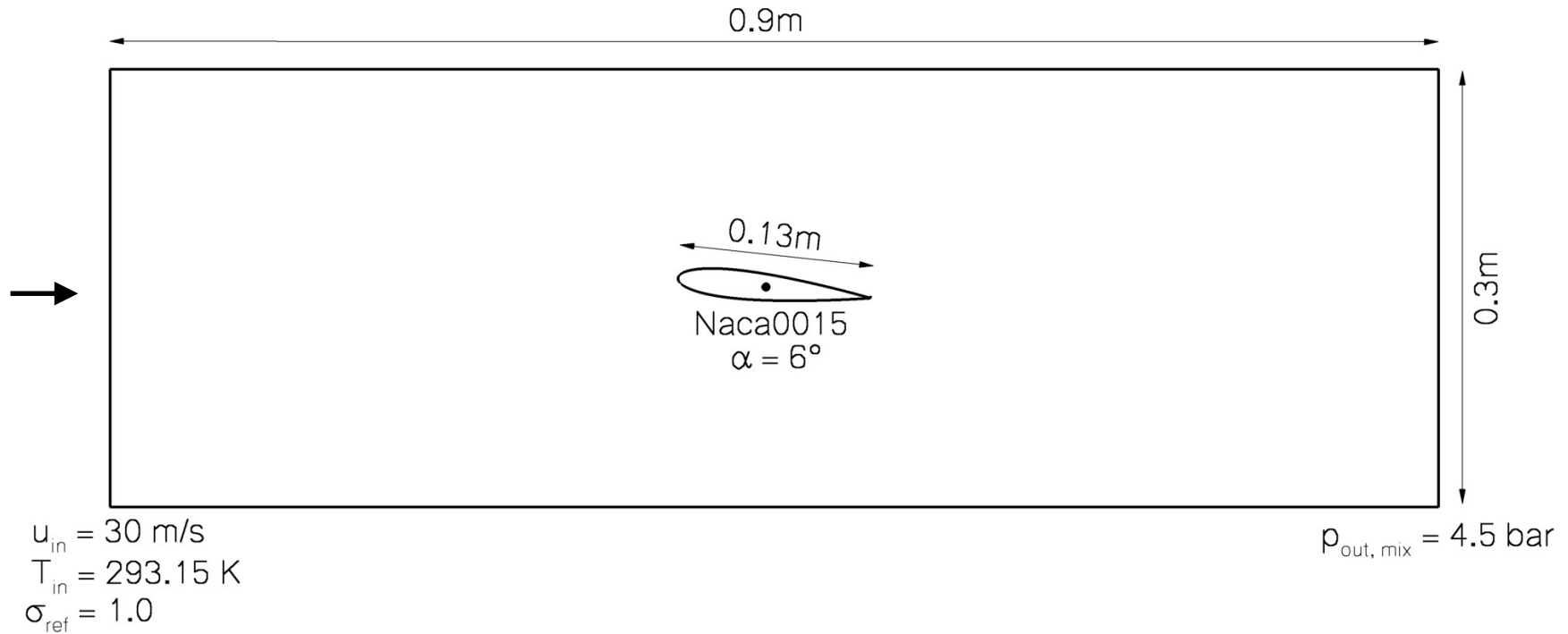


**Experiment:** Brandner, P. A., Walker, G. J., Niekamp, P. N. and Anderson, B., “An Investigation of Cloud Cavitation about a Sphere.” In: 16th Australasian Fluid Mechanics Conference, 2 – 7 December 2007, Crown Placa, Gold Coast, Australia, 2007.



**Simulation CATUM:** Isosurfaces  $\alpha=0.05$ , one instant in time.

## Numerical results – 2-D hydrofoil



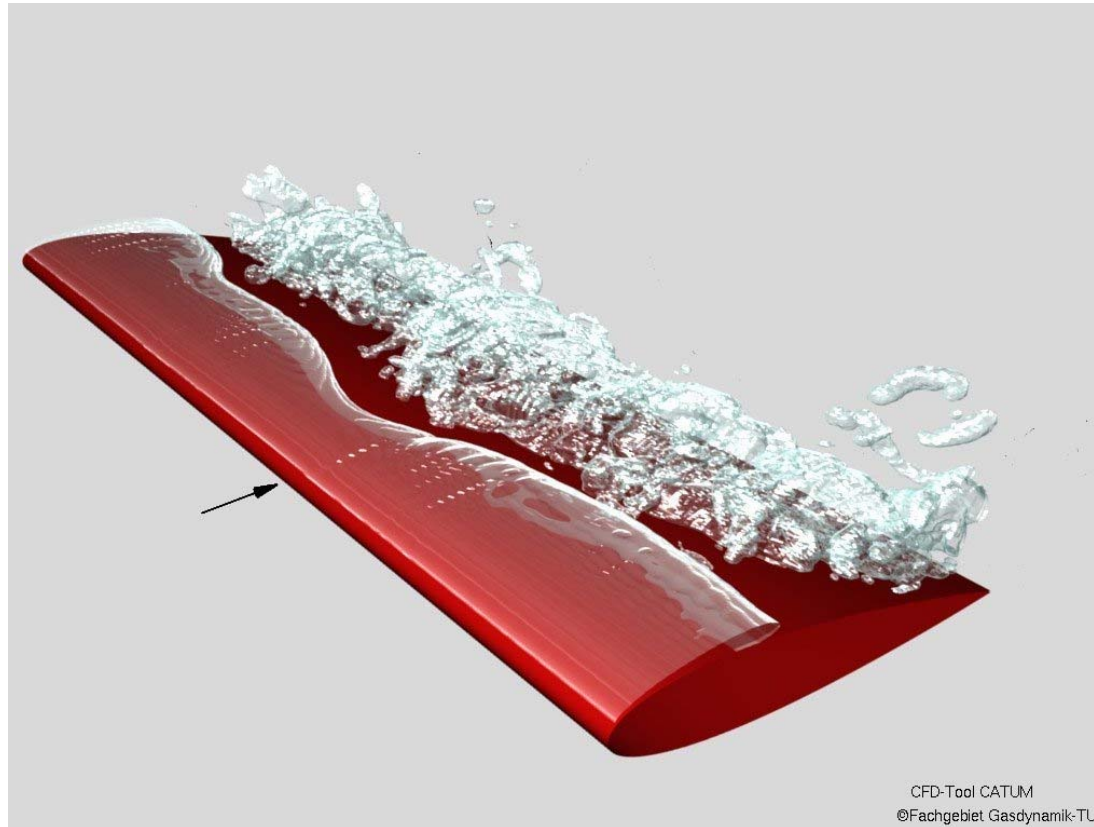
### 3-D simulation – span/channel width 0.3 m

$2.4 \cdot 10^7$  cells

$6 \cdot 10^5$  time steps,  $\Delta t \approx 5 \cdot 10^{-8} \text{ s}$ ,



## Numerical results – 2-D hydrofoil



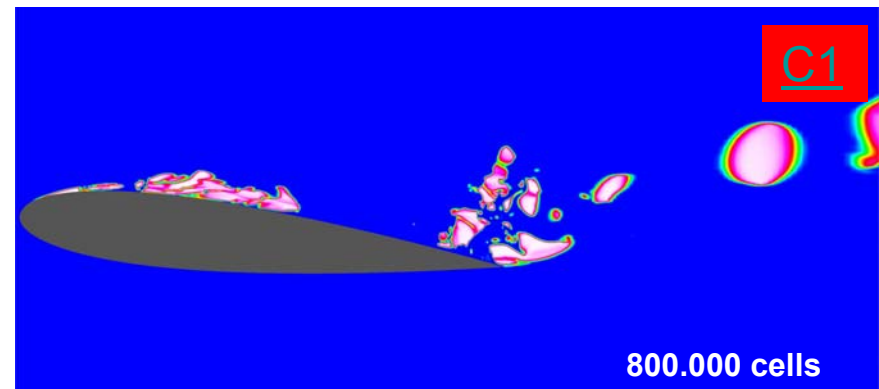
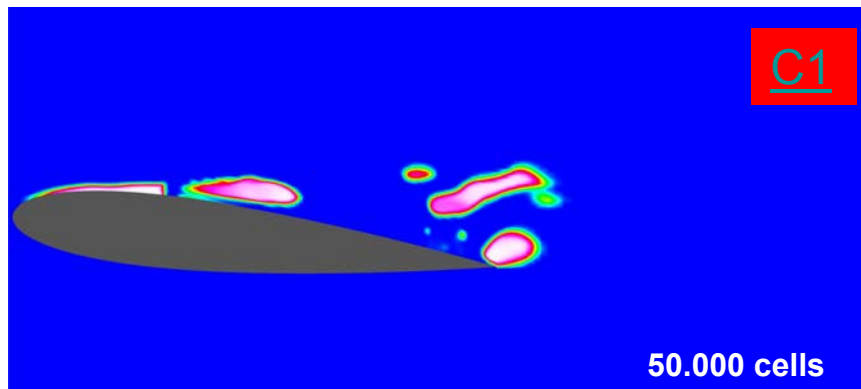
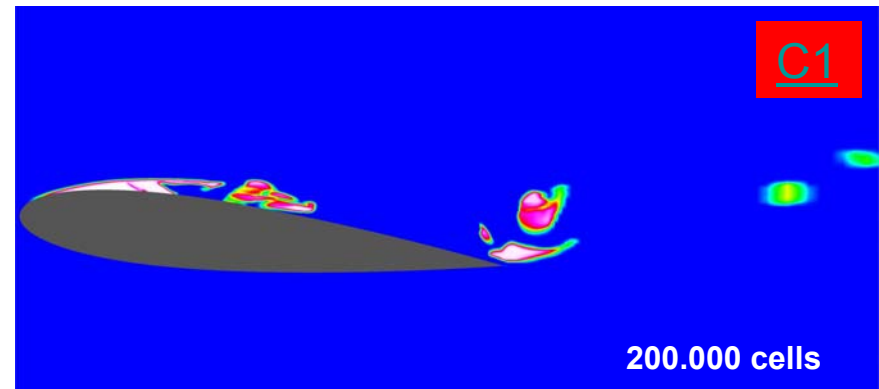
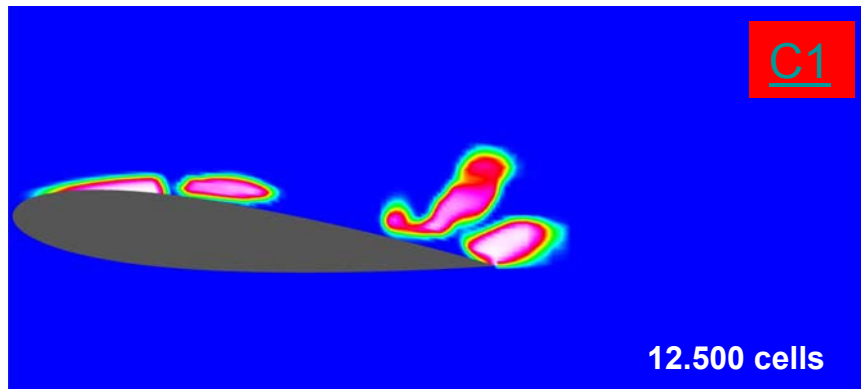
$$f_{\text{zyklus}} = 100 \text{ Hz,}$$

$$\Delta t_{\text{movie}} = 3 \cdot 10^{-2} \text{ s}$$

$2.4 \cdot 10^7$  cells,  
 $6 \cdot 10^5$  time steps,  $\Delta t \approx 5 \cdot 10^{-8} \text{ s}$ ,  
 96/192 CPU (lx64a Opteron)  $\rightarrow$  500 h

## Numerical results – fragmentation of two-phase flow

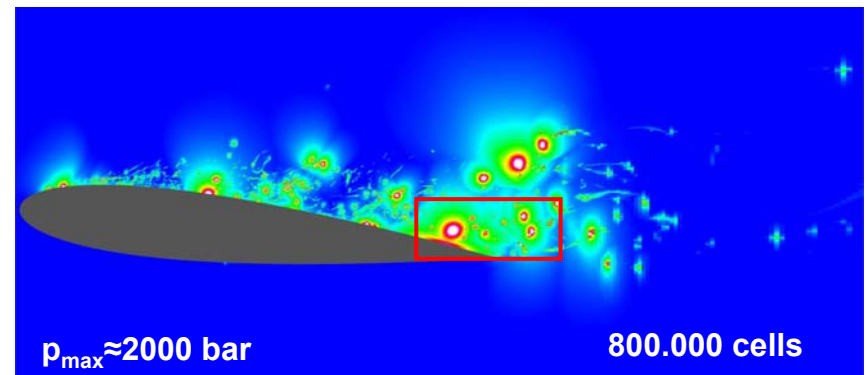
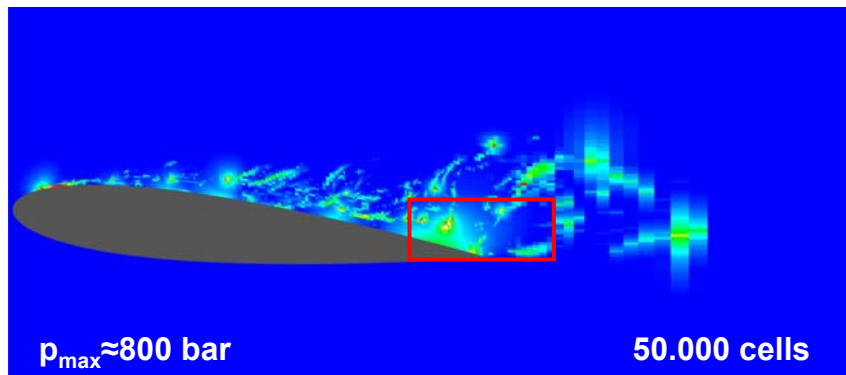
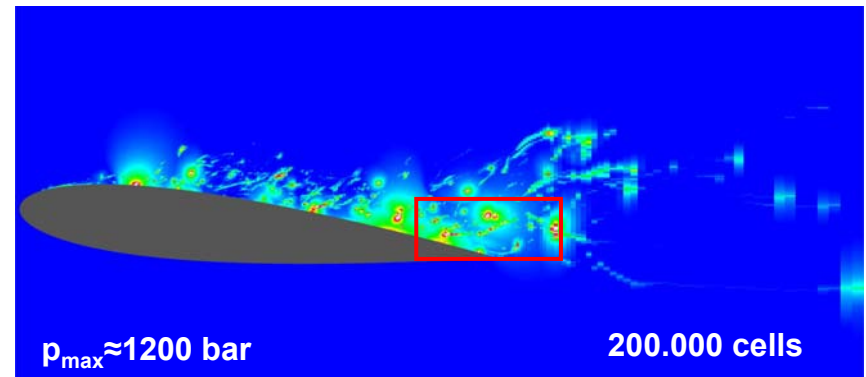
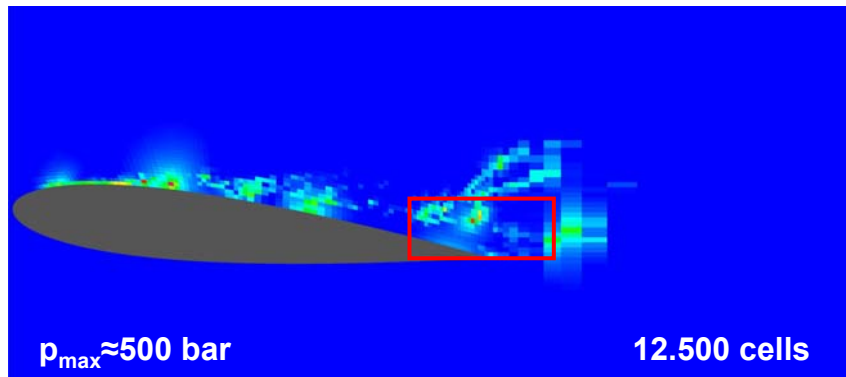
2-D cavitation on 2-D hydrofoil



Effect of the spatial resolution on the structures of the vapor volume fraction  $\alpha$

## Numerical results – fragmentation of two-phase flow

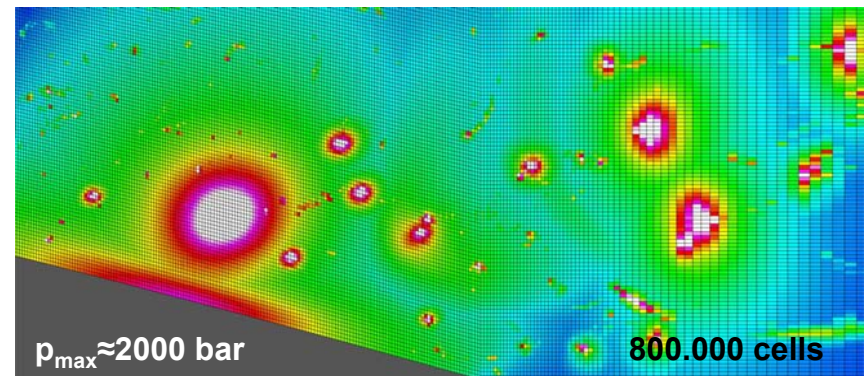
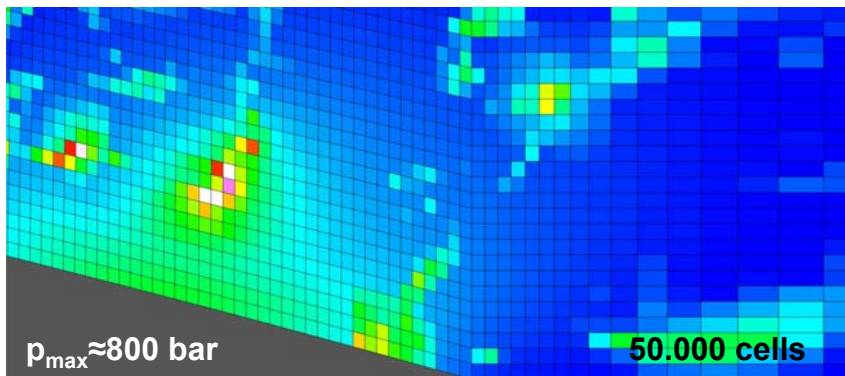
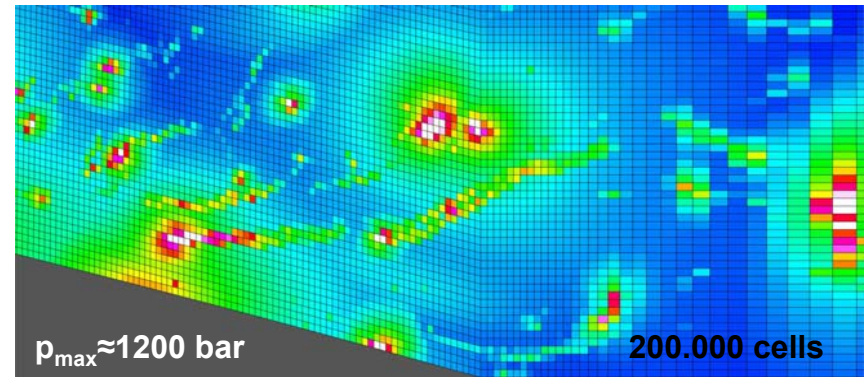
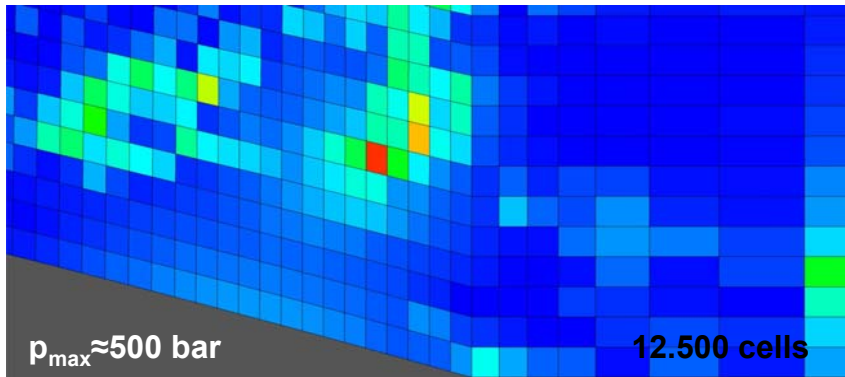
### 2-D cavitation on 2-D hydrofoil



Effect of the spatial resolution on the instantaneous maximum loads –  
 pressure footprint over one cycle

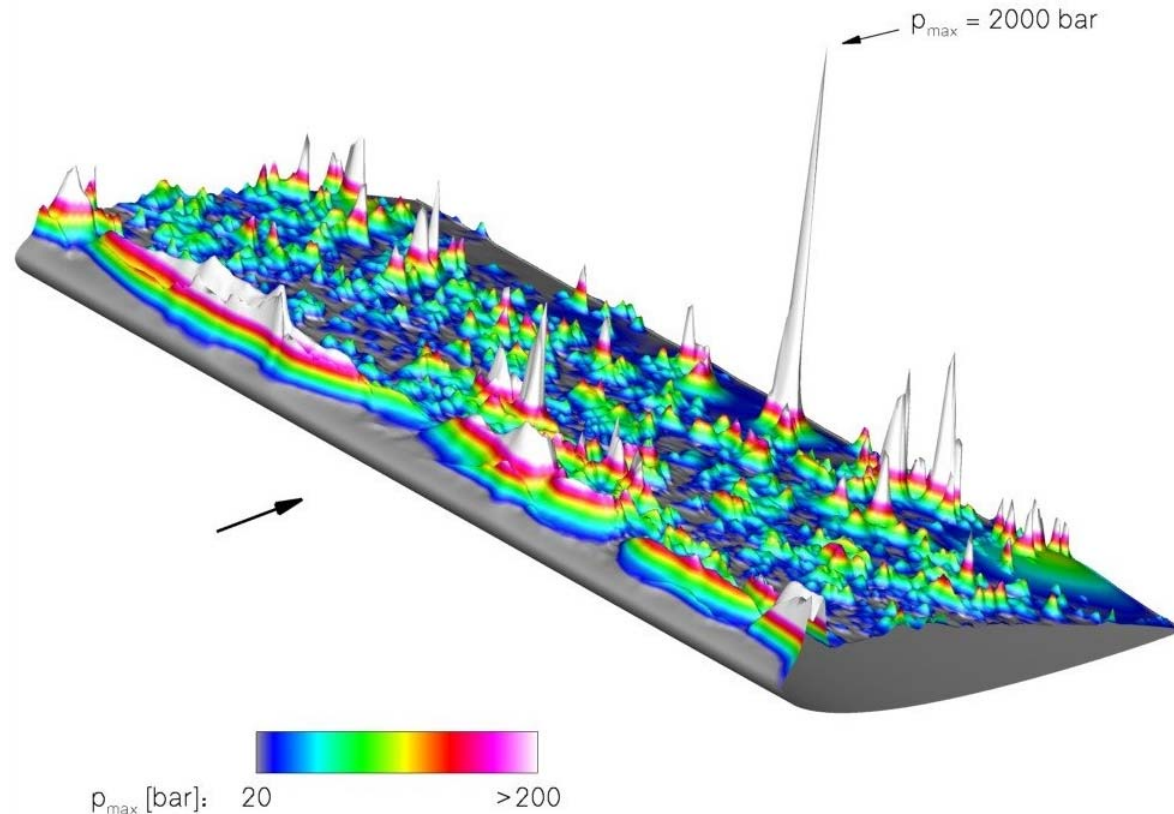
## Numerical results – fragmentation of two-phase flow

### 2-D cavitation on 2-D hydrofoil



Effect of the spatial resolution on the instantaneous maximum loads –  
 pressure footprint over one cycle, **zooms of previous pictures**

## 2-D hydrofoil – maximum pressure



Collaps induced maximum pressure on the suction side -  $p_{\max} \approx 2000 \text{ bar}$

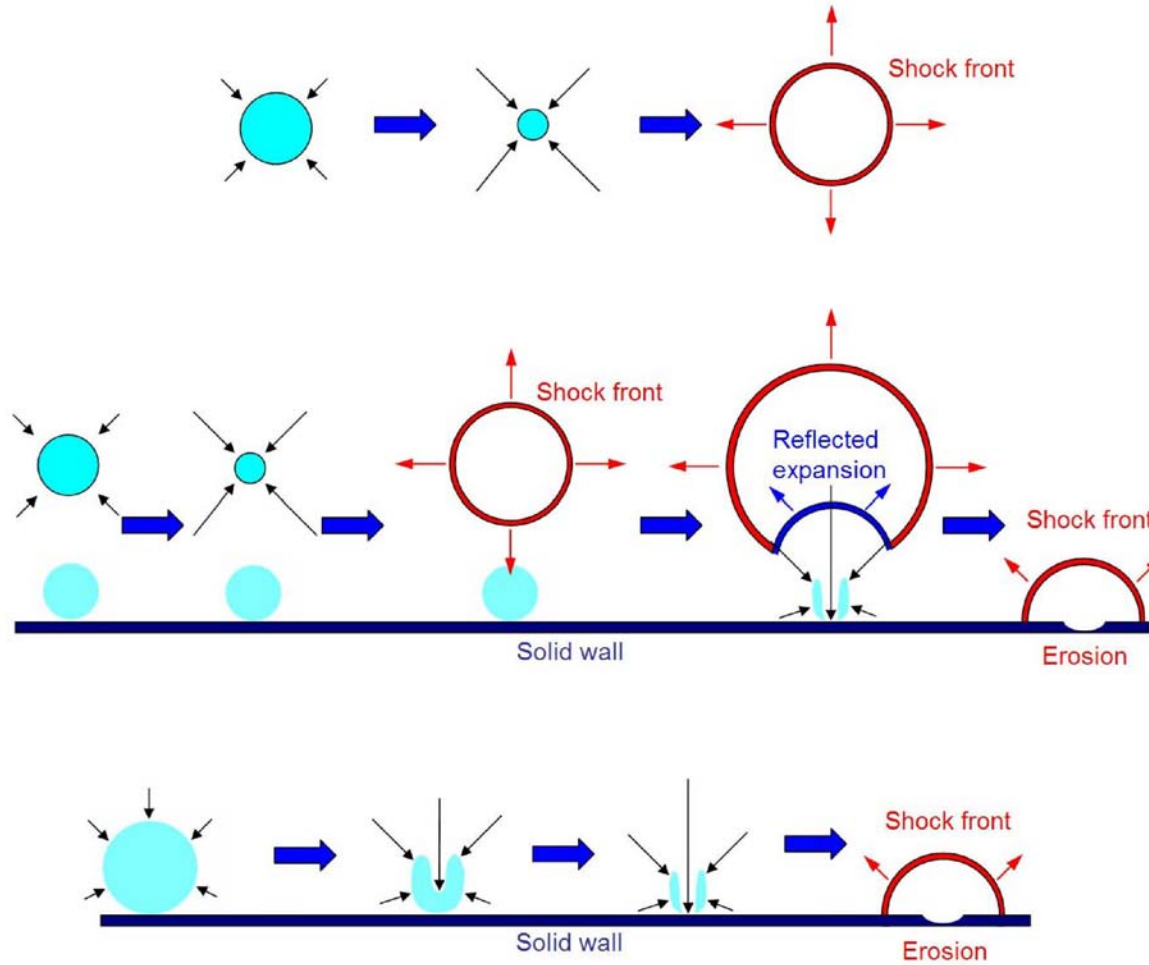
**Analysed time: one period with  $\Delta t_{\text{zyklus}} = 10^{-2} \text{ s}$**

## Cavitation erosion

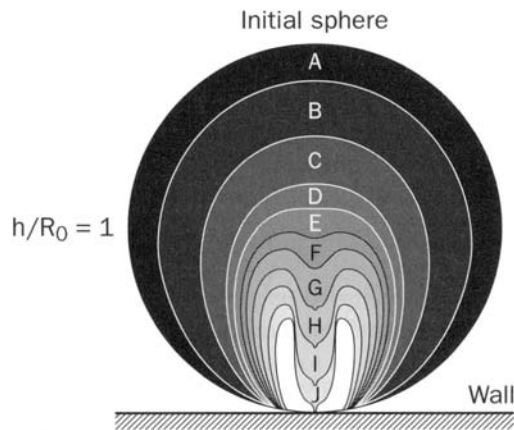


Kuiper, G. - **MARIN** Maritime Research Institute - The Netherlands

## Driving mechanisms of cavitation erosion

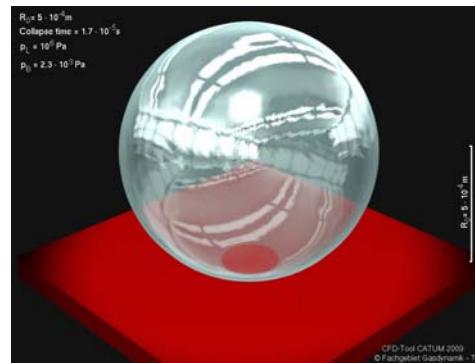


## Single bubble collapse with wall interaction

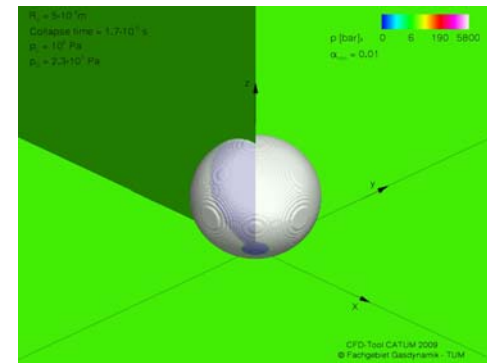


J.P. Franc, J.M. Michel:  
 „Fundamentals of Cavitation“, 2004

**M4**



**M5**



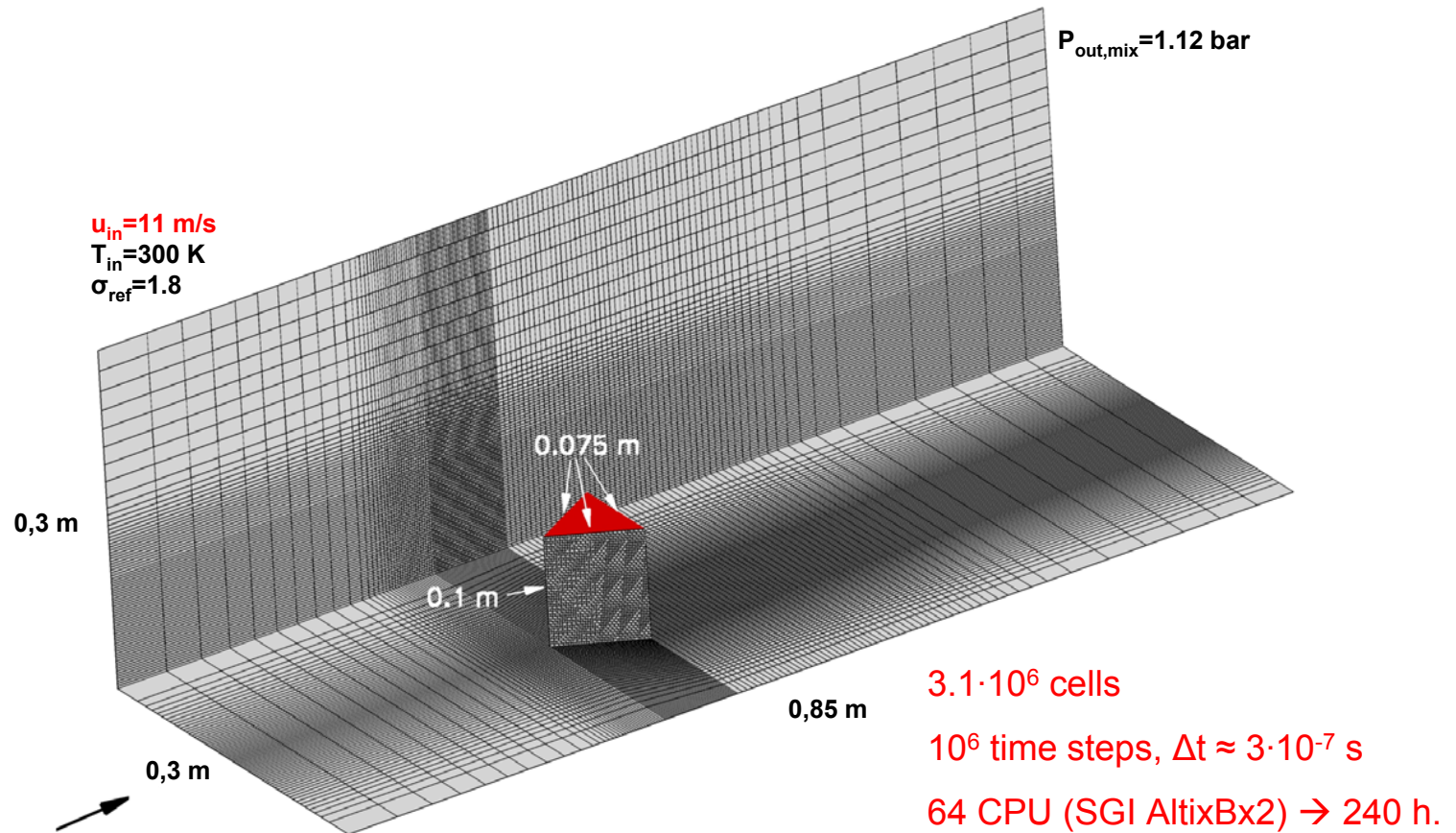
Simulation **CATUM**

Initial radius  $R_0=0.5$  mm, time step  $\Delta t_{CFD}=6.0 \cdot 10^{-9}$  s, collapse time  $1.7 \cdot 10^{-5}$  s,  
 Initial pressures  $p_{liquid}=10.0$  bar,  $p_{bubble}=0.023$  bar ,  $T=293$  K, water/vapor



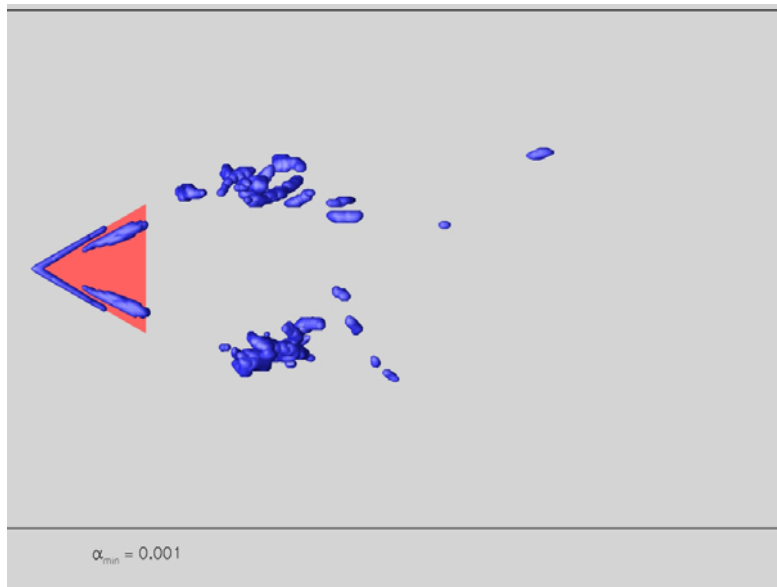
## Numerical results – Erosive two-phase flow

3-D simulation of the “Obernach-experiment” on **cavitation erosion**

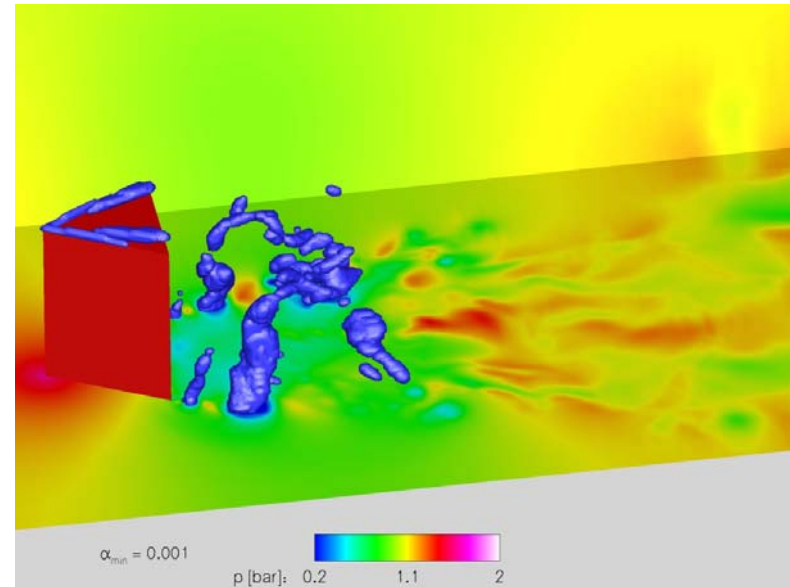


## Numerical results – Erosive two-phase flow

Dynamic phase-transition and related pressure field



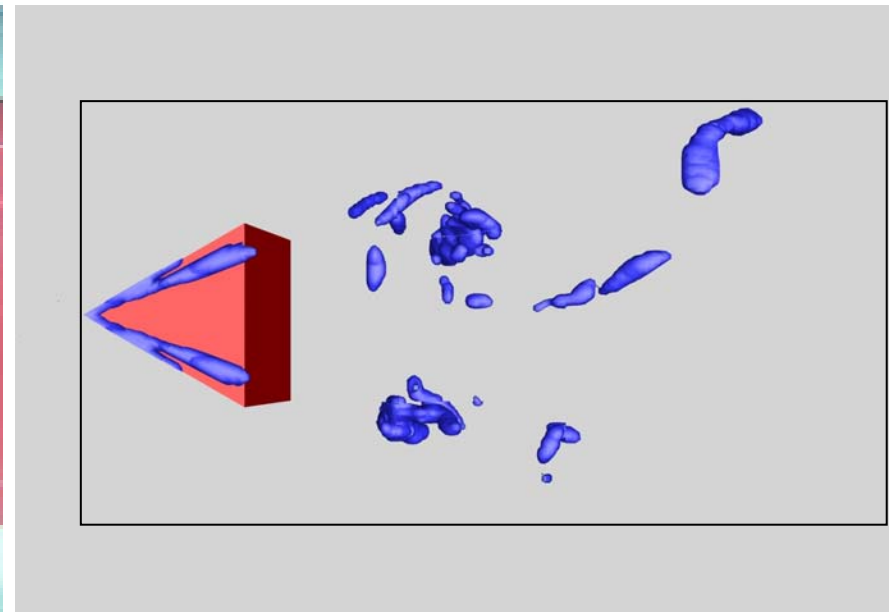
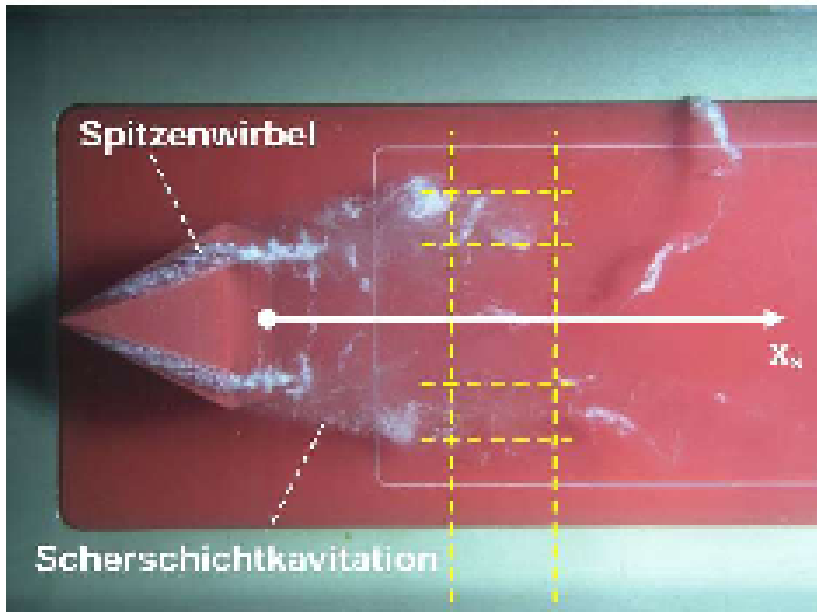
Top view: Two-phase regions,  $\Delta t_{\text{Movie}} = 0.17$  s.



Perspective view: Two-phase regions and static pressure at the walls,  $\Delta t_{\text{Movie}} = 0.17$  s.

## Numerical results – Erosive two-phase flow

Comparison of two-phase structures experiment/simulation

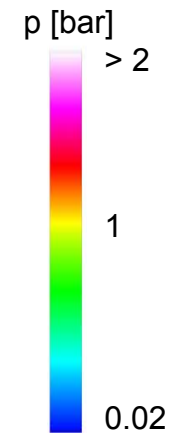
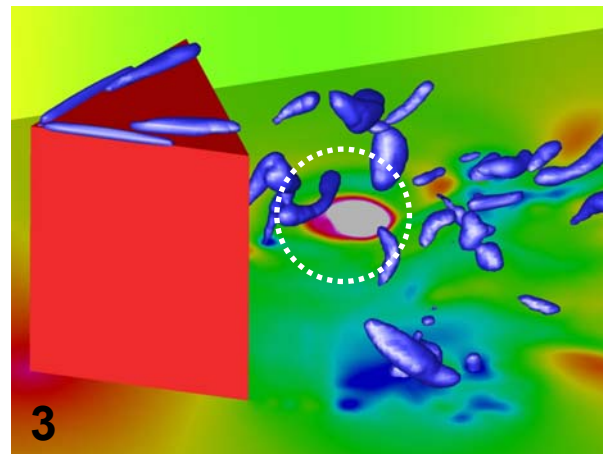
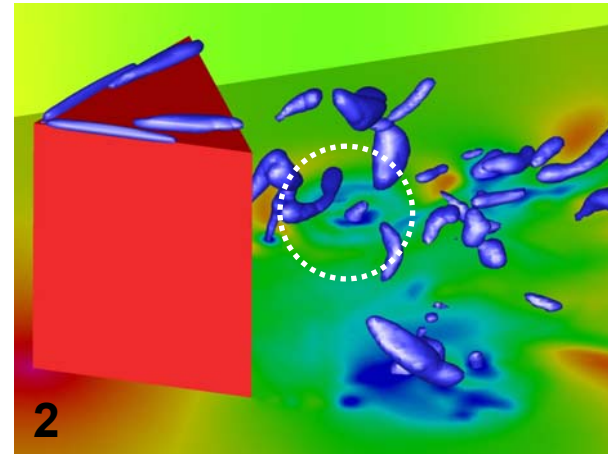
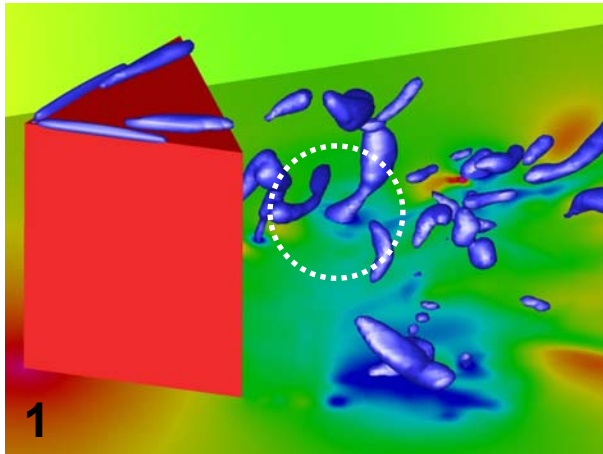


**Experiment:** Huber R., Geschwindigkeitsmaßstabseffekte bei der Kavitationserosion in der Scherschicht nach prismatischen Kavitatoren, Berichte des Lehrstuhls und der Versuchsanstalt für Wasserbau und Wasserwirtschaft, Hrsg. Univ.-Prof. Dr.-Ing. Th. Strobl, Nr. 102, 2004.

**Simulation CATUM:** Isosurfaces  $\alpha=0.01$ , one instant in time.

## Numerical results – Erosive two-phase flow

Fragmentation of two-phase structure, collapse, shock formation



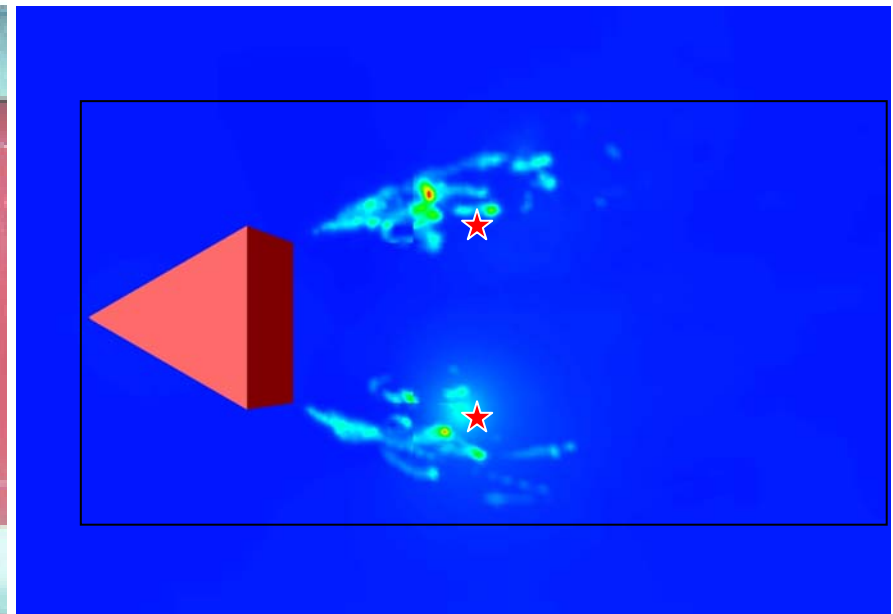
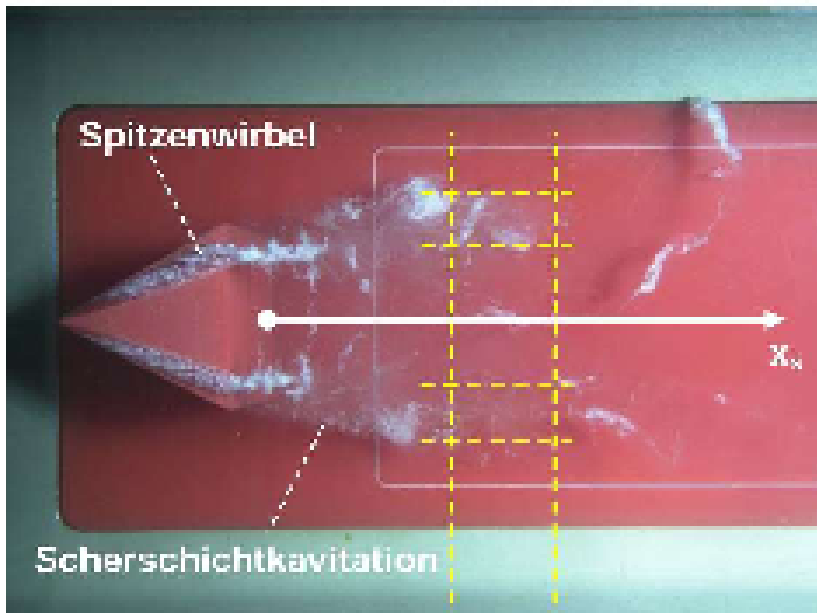
$$\Delta t_{1 \rightarrow 2} = 1.17 \cdot 10^{-4} \text{ s}$$

$$\Delta t_{2 \rightarrow 3} = 0.58 \cdot 10^{-4} \text{ s}$$

$$p_{\max} = 65 \text{ bar}$$

## Numerical results – Erosive two-phase flow

Areas of intense erosion (experiment) - maximum pressures (simulation)



**Experiment:** Huber R., Geschwindigkeitsmaßstabeffekte bei der Kavitationserosion in der Scherschicht nach prismatischen Kavitatoren, Berichte des Lehrstuhls und der Versuchsanstalt für Wasserbau und Wasserwirtschaft, Hrsg. Univ.-Prof. Dr.-Ing. Th. Strobl, Nr. 102, 2004.

**Simulation CATUM:** Collapse induced maximum pressure at the bottom wall of the numerical test-section, analysis interval **0.058 seconds**. Stars indicate the barycenters (experimental) of the erosion areas.

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# Discussion