Der Satz von Immerman-Szelepcsényi

Sommerakademie Rot an der Rot — AG 1
Wieviel Platz brauchen Algorithmen wirklich?

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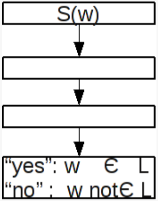
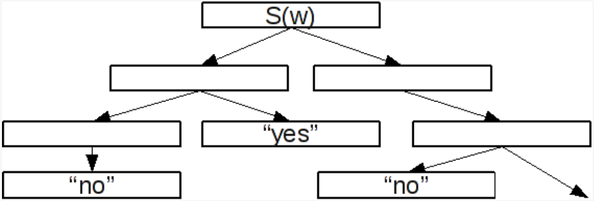
Motivation

- 1964 Kuroda: $ContextSensitive = NLINSPACE = NSPACE(O(n))$
- 1. LBA Problem: $DSPACE(n) = NSPACE(n)$?
  still open, Theorem of Savitch
- 2. LBA Problem: $\bar{L} := \Sigma^* \setminus L$ also context sensitive?
  1988 Immermann:
  "Nondeterministic space is closed under complementation."
  1988 Szelepcsényi:
  "The Method of Forced Enumeration for Nondeterministic Automata."

2. LBA Problem: Description $w \notin L$

- Deterministic vs. nondeterministic calculation tree





- $f(n)$-Space bounded TM: Graph with at most $c^{f(n)}$ nodes
- How to insure that there exists no "yes"-node along the reachables from $S(w)$ without loosing the space bound?
- Idea: Time consumption is totally irrelevant
  Save space through ad-hoc recalculation

Outline

Notation

- TM $M = (Q, \Sigma, \Gamma, \delta, q_0, F, \Box)$
  Q finite state set, $q_0 \in Q$ initial state, $F \subseteq Q$ set of final states,
  Σ input alphabet, Γ tape alphabet, $\Box \in \Gamma \setminus \Sigma$ blank symbol,
  $\delta \subseteq Q \times (\Sigma \cup \{\Box\}) \times \Gamma^k \times Q \times \Gamma^k \times \{L, R, N\}^{k+1}$
- Configuration / snapshot of a TM: $\alpha, \beta, \cdots$
  - $\alpha = (i, q, w_1, \cdots, w_k)$ k-tape TM
  - $\alpha = a_1 \cdots a_{i-1} q a_i \cdots a_n$ 1-tape TM
- start configuration $S(w) := q_0 w$
- $\alpha \vdash_M^1 \bar{\alpha}$, $\alpha \vdash_M^k \bar{\alpha}$
- Length of a configuration: $|\alpha| = |a_1 \cdots a_{i-1}| + |a_i \cdots a_n|$
- $NSPACE(f(n)) =$
  $\{L | L$ accepted by nondet. TM, longest configuration $\leq f(n)\}$
- $coNSPACE(f(n)) = \{\bar{L} := \Sigma^* \setminus L | L \in NSPACE(f(n))\}$

Theorem (Immermann Szelepcsényi)

Let M TM with $L(M) \in NSPACE(f)$, $f \in \Omega(log(n))$
There exists a nondet. TM M' with $L(M') = \bar{L}$ and

$$L(M') \in NSPACE(O(f)).$$

- NSPACE is closed unter complement: $NSPACE(f) = coNSPACE(f)$

Preobservations

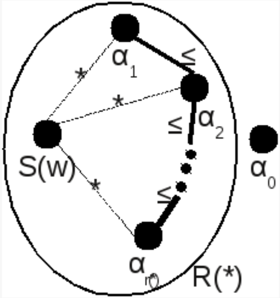
- M is a 1-tape TM (sec. 4)
- Configurations are strings, easy enumerable
- $Unreach(Graph, S(w), "yes")$?
- Nondeterminism allows guess & check
- Need of the count of different strings to be guessed
- Check if "yes"-string is contained.

General simplifications

- $\leq$ order on configurations (e.g. length lexicographic)
- "*yes*" $= \alpha_0$ is shortest and only accepting configuration
- $R(k) = \{\alpha | S(w) \vdash_M^{\leq k} \alpha\}$ reachables
- $r(k) = |R(k)|$
- $r(*) = r(\infty) \leq c^{f(|w|)}$

M': Unreach

Data: $w \in \Sigma^*$
Result: "$w \notin L$" or *stop*
$\alpha := \alpha_0$;
for $r(*)$ **do**
  Guess: $S(w) \vdash_M^* \bar{\alpha}$;
  if $\bar{\alpha} \leq \alpha$ **then**
    stop;
  else
    $\alpha := \bar{\alpha}$;
  end
end
return "$w \notin L$";
Algorithm 1: Unreach



- $w \notin L$: $R(*) = \{\alpha_1, \alpha_2, \cdots, \alpha_{r(*)}\}$ and $\alpha_i \leq \alpha_{i+1}$.
- $w \in L$: $\alpha_0 \in R(*) => stop$.

Data: $S(w)$
Result: $r(*)$ or *stop*
$r(0) := 1;$
$m(0) := |S(w)|;$
$k := 0;$
while $r(k) \neq r(k+1)$ **do**
    $k := k + 1;$
end
return $r(k)$;

Algorithm 2: $r(*)$

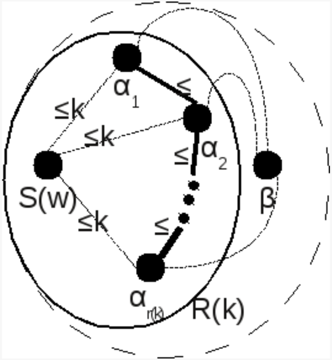
Data: $r(k), m(k) \in \mathbb{N}$
Result: $r(k+1), m(k+1)$ or *stop*
$\alpha := \alpha_0;$
$r := 0;$
$m := 0;$
for $\beta : |\beta| \leq m(k) + 1$ **do**
    if $\beta \in R(k+1)$ **then**
        $r := r + 1;$
        $m := max\{m, |\beta|\};$
    end
end
return r, m;

Algorithm 3: Inductive counting

Data: β: Konfiguration, $k \in \mathbb{N}$
Result: "$\beta \in R(k+1)$" or *stop*

$\alpha := \alpha_0$;
$b :=$ *false*;
for $r(k)$ **do**
  Guess: $S(w) \vdash_M^{\leq k} \bar{\alpha} \in R(k)$;
  if $\bar{\alpha} \leq \alpha$ **then**
    stop;
  else
    $\alpha := \bar{\alpha}$;
  end
  if $\bar{\alpha} \vdash_M^{\leq 1} \beta$ **then**
    $b :=$ *true*;
  end
end
return *b*;

Algorithm 4: $\beta \in^? R(k+1)$



Space consumption of M'

- check $\alpha \leq \beta$ needs at most $2 \cdot f(n)$
- 6 configurations as local variables
- $k, m \leq log(c^{f(n)}) \in O(f(n))$
- increment is easy with TM
- check $\alpha \vdash_M^{\leq 1} \beta$ in $2 \cdot f(n)$

$L(M') \in NSPACE(O(f(n)))$

Consequences

- 2. LBA Problem: is $\Sigma^* \setminus L_1$ of type 1 ? Yes
- Context sensitive languages are closed under complement.

How to Simulate a k-tape TM with 1 tape?

Aim: Simulation of k-tape TM M on a 1-tape TM M'

Idea: Alphabet extention to tuples

- Let $M = (Q, \Sigma, \Gamma, \delta, q_0, F, \Box)$ k-tape TM
- $M' := (Q', \Sigma, (\Gamma \cup \hat{\Gamma})^k, \delta', q_0', F', \Box)$
- verbal description of δ'
  - copy input $w = a_1 \cdots a_n$ to tupels $(a_1, \hat{\Box}, \cdots, \hat{\Box})(a_2, \Box, \cdots, \Box) \cdots (a_n, \Box, \cdots, \Box)$
  - for q at a_i search sequentially for head positions
  - save head letters in state
  - perform replacement of δ

Thank you for paying attention!

Do you have questions?

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