Diskrete Stammfunktion – Beispiele

1. $\Delta \underbrace{x^{\underline{m+1}}}_{f(x)} = \underbrace{(m+1)\, x^{\underline{m}}}_{g(x)}$ $\qquad \left(\frac{d}{dx}(x^n) = n \cdot x^{n-1}\right)$

$m \neq -1$.

$$\sum_{y=0}^{x-1} y^{\underline{m}} = \frac{1}{m+1} \cdot \sum (m+1) \cdot y^{\underline{m}} \underset{\substack{\uparrow \\ \text{Satz}}}{=} \frac{((x-1)+1)^{\underline{m+1}}}{m+1} + 0$$

Analysis: $\displaystyle\int_a^b \frac{d\, f(x)}{dx}\, dx = f(b) - f(a)$

$$\frac{d}{dx}\left(\frac{x^{m+1}}{m+1}\right) = x^m, \qquad \int x^m\, dx = \frac{x^{m+1}}{m+1} + C$$

2. $f(x) := \sum x^{\underline{-1}}$ heißt $g(x) := x^{\underline{-1}}$, also

$$f(x+1) - f(x) = \Delta f(x) = g(x) = x^{\underline{-1}} = \frac{1}{x+1}$$

Vergleiche H_x mit $\log x$ $\left(\approx \int \frac{dx}{x}\right)$

3. Allgemein: $f = \sum g \simeq \Delta f = g$

4. $x^{\underline{2}} + x^{\underline{1}} = x \cdot (x-1) + x = x(x-1+1) = x^2$

Wegen $\displaystyle x^m = \sum_{k=0}^{m} S_{m,k} \cdot x^{\underline{k}}$

kann jedes Polynom in der Form

$\displaystyle\sum_{i=0}^{n} b_i\, x^{\underline{i}}$ geschrieben werden.

$$\Delta^i x^{\underline{k}} = \Delta^{i-1}(\Delta x^{\underline{k}}) = \Delta^{i-1}(k \cdot x^{\underline{k-1}}) = k^{\underline{i}} \cdot x^{\underline{k-i}}$$

Partielle Summation:

$$f \cdot g = \sum (f \cdot \Delta g) + \sum ((Eg) \cdot \Delta f)$$

vergleiche partielle Ableitungen.

$$\sum (f \cdot \Delta g) = f \cdot g - \sum ((Eg) \cdot \Delta f)$$

<u>Beisp:</u> $\Delta g = \Delta \binom{x}{m+1} = \binom{x}{m}$,

$$\sum \Delta g = \sum \binom{x}{m} = \binom{x}{m+1}, \quad \Delta H_x = \frac{1}{x+1}$$

$$\Delta^k x^n = \sum_{i=0}^{k} (-1)^{k-i} (x+i)^n$$

$$v_n = \sum_{k=0}^{n} \binom{n}{k} \underbrace{D_k}_{u_k} = n! \Leftrightarrow D_n = \sum_{k=0}^{n} (-1)^{n-k} \binom{n}{k} \underbrace{k!}_{v_k}$$

Analysis: $e^x = 1 + \frac{x}{1!} + \frac{x^2}{2!} + \dots + \frac{x^n}{n!} + \dots$

$$\frac{1}{1-z} = \sum_{n \geq 0} z^n \;\Big/\; \frac{1}{1+z},\ \frac{1}{1-z^2},\ \frac{1}{1-az}$$

Analysis: $(1+x)^y = \sum_{k \geq 0} \binom{y}{k} x^k$

für $x, y \in \mathbb{N}$: binomische Formel

$$(-c)^{\underline{n}} = (-c-1)(-c-2) \dots (-c-n+1)$$

$$(c+n-1)^{\underline{n}} = (c+n-1)(c+n-2) \dots (c+2)(c+$$

$$\sum \binom{x+1}{m+1} \cdot \frac{1}{x+1} = \sum \frac{(x+1)\, x^{m}}{(m+1)\, m!} \cdot \frac{1}{x+1}$$

$$f(x) = \sum_{i=0}^{n} a_i x^i$$

$$= \sum_{i=0}^{n} f_i\, p_i(x)$$

$$p_i(x) = p_{ii} x^i + p_{i\,i-1} x^{i-1} + \dots p_{i0}$$

$$= \sum_{i=0}^{n} f_i \left(p_{ii} x^i + p_{i+1\,i} x^i + p_{i+2\,i} x^i + \dots \right)$$

Beisp: $P_0 = 1$, $P_1 = 2x+1$, $P_2 = 3x^2 + 3x + 2$

$P_3 = x^3 + x^2 - x + 4$

$$f(x) = \underline{2}\, x^3 + \underline{5}\, x^2 + \underline{3}\, x + 2 = \sum_{i=0}^{3} f_i\, p_i$$

$$= f_3 (x^3 + x^2 - x + 4) +$$

$$f_2 (3x^2 + 3x + 2) + f_1 (2x+1) + f_0 \cdot 1 =$$

$$= \underline{f_3}\, x^3 + (\underline{f_3 + f_2 \cdot 3})\, x^2 + (\underline{f_3 (-1) + f_2 \cdot 3 + f_1 \cdot 2})\, x$$

$$+ f_3 \cdot 4 + f_2 \cdot 2 + f_1 \cdot 1 + f_0 \cdot 1$$

Koeffizientenvergleich:

$\underline{f_3 = 2}$, $f_3 + 3 f_2 = 5 \Rightarrow 3 f_2 = 3$, $\underline{f_2 = 1}$,

$-f_3 + 3 f_2 + 2 f_1 = 3 \Rightarrow -2 + 3 + 2 f_1 = 3$, $\underline{f_1 = 1}$,

$4 f_3 + 2 f_2 + f_1 + f_0 = 2 \Rightarrow 8 + 2 + 1 + f_0 = 2, \dots$