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Theory of Network Communication

Fall 2002

Midterm Exam

Problem 1: Basic Understanding (5 points)
Please answer these questions in words (not calculations).

a) Why is the flow number a useful network parameter?

b) How do we have to manipulate a hypercube to obtain a butterfly?

c) Why is it in general not a good idea to use a path system for oblivious routing that has only a single path for every source-destination pair?

d) Does it suffice for a protocol to guarantee that every packet injected at time t with a path of length d has a delay of $d \cdot t$ to be stable?

e) In which case can a directed graph with at most one edge for every pair of nodes and that contains two directed cycles be universally stable?

Problem 2: Network Theory (5 points)
For any $d \in \mathbb{N}$ let $G_d = (V, E)$ be a graph with node set $V = \{1, 2, \ldots, 2^d - 1\}$ and edge set $E = \{\{v, 2 \cdot v\}, \{v, 2 \cdot v + 1\} \mid v \in \{1, \ldots, 2^{d-1} - 1\}\}$.

a) What is the degree of G_d? Justify your answer. (2 points)

b) What is the diameter of G_d? Justify your answer. (3 points)

Hint: it may help to use a binary representation for the nodes.

Problem 3: Network Simulation (4 points)
Consider the approach of embedding a hypercube into an $n \times n$-mesh of the same number of nodes by mapping the nodes of the hypercube one-to-one to nodes in the mesh. If we want to simulate a communication step of the hypercube, we have to be able to send in the worst case a packet (or equivalently, a demand of 1) for every pair of nodes in the mesh that represents a pair of nodes in the hypercube that is connected by an edge. To come up with good paths for this, we have to solve a suitable concurrent multicommodity flow problem.

a) What is the total demand leaving and leading to any node of the mesh for this multicommodity flow problem (we assume that the hypercube edges are bi-directional)? (1 point)

b) Argue, how one can obtain a solution for this multicommodity flow problem with congestion $O(n \log n)$ and dilation $O(n)$. You can cite results from the lecture notes. (3 points)

Problem 4: Universal stability of queueing disciplines (6 points)
Suppose we have packets with two different priority levels. High-priority packets are always preferred against low-priority packets. For packets within a priority class we use SIS to decide which to prefer. Our aim is to (partly) show that such a queueing discipline is universally stable. Suppose that the maximum length of a path selected by the adversary is D.

a) First, give an upper bound on the routing time of a high-priority packet. (1 point)

b) Suppose that the upper bound for a) is T_1. Argue that in this case there can be at most $(T_1 + T_2 + w)\lambda$ packets that can prevent a low-priority packet from crossing its first edge for T_2 steps. (2 points)

c) Use a) and b) to prove an upper bound on the delay of a low-priority packet at its first edge in terms of w, λ, and D. (3 points)

Good luck!