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Baltimore, Oct. 20 2003

Theory of Network Communication

Fall 2002

Midterm Exam

Problem 1: Basic Understanding (5 points)
Please answer these questions in words and/or pictures (not calculations).

(a) Suppose we have a maximum flow problem with multiple sources and destinations, and every source can send flow to any destination. Does it suffice to use a single-commodity flow algorithm, or do we need a multi-commodity flow algorithm to solve this problem? Justify your answer.

(b) How do we have to manipulate a hypercube to obtain a butterfly?

(c) What is the difference between oblivious and adaptive routing?

(d) Why is it in general not a good idea to use a path system for oblivious routing that has only a single path for every source-destination pair?

(e) Is there a difference in the performance of the FIFO queueing rule between the adversarial and the stochastic injection model? Why?

Problem 2: Degree and Diameter (4 points)
For any $d \in \mathbb{N}$ let $G_d = (V, E)$ be a graph with node set $V = [2]^d$ and edge set $E = \{\{x, y\} \mid x = (x_{d-1}, \ldots, x_0)$ with $x_{d-1} = 0$ and $y = (x_{d-2}, \ldots, x_0, y_0)$ with $y_0 \in \{0, 1\}\}$.

a) What is the degree of G_d? Justify your answer. (2 points)

b) What is the diameter of G_d? Justify your answer. (2 points)

Hint: it may help to draw a picture of G_d for small d, starting with node $(0, \ldots, 0)$.

Problem 3: Expansion (2 points)
Compute the expansion of a d-dimensional hypercube. It is sufficient here to guess the right set U and compute the value $c(U, \bar{U})/\min\{c(U), c(\bar{U})\}$.

Problem 4: Multicommodity Flows (3 points)
Consider the discrete Awerbuch-Leighton algorithm given in assignment 3 (see also Figure 1) for $(1 + \epsilon)$-feasible multicommodity flow problems of demand 1 for every commodity, i.e. the demands can be increased by a $(1+\epsilon)$ factor and there is still a feasible solution. Use the fact that it guarantees bounded buffers to design a discrete Awerbuch-Leighton algorithm with bounded

buffers for $(1+\epsilon)$-feasible multicommodity flow problems of arbitrary integral demands. (A proof of correctness is not needed.)
Hint: a commodity of demand d can be seen as d commodities of demand 1. (However, the algorithm has to be expressed in terms of the original commodities!)

What about $(1+\epsilon)$-feasible multicommodity flow problems with arbitrary positive demands? (2 extra points)

Discrete Awerbuch-Leighton Algorithm:
At each node u:

1. Distribute newly injected flow evenly among the buffers $Q_i(e)$, i.e. distribute it so that afterwards for every i, $\bar{q}_i(e)$ is the same for every edge e leaving u.
2. For every edge (u, v), select any i with maximum $\Delta_i(u, v)$. If this is negative, no flow is sent. Otherwise, compute $f_i = \min\{1,\ \Delta_i(u, v)/2\}$ and send a flow of $c(e) \cdot f_i$ from $Q_i(u, v)$ to $Q_i(v, u)$.
3. Receive the transmitted flow and absorb flow that reached its destination.
4. Rebalance the queue heights so that for every i, $\bar{q}_i(e)$ is the same for every edge e leaving u.

Figure 1: The Discrete Awerbuch-Leighton algorithm.

Problem 5: Routing (4 points)

(a) Consider the problem of routing a multicommodity flow problem in an $n \times n$-mesh in which the total demand leaving or leading to a node is at most d. Use the fact that the flow number of the $n \times n$-mesh is $\Theta(n)$ to compute an upper bound on the congestion and dilation (in terms of n and d) of routing any such multicommodity flow problem. (2 points)

(b) Consider the problem of simulating a communication step of a hypercube by a mesh of the same size. For this, we may use any one-to-one mapping of the hypercube nodes to the mesh nodes. Use [(a)] to determine the congestion and dilation of simulating an arbitrary communication step in the hypercube (i.e. along each edge a flow of up to 1 may be sent) by the mesh. (2 points)

Good luck!