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Problem Sheet Model Test
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Fundamental Algorithms - Surprise Test

Name:
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Problem 1 (20 Points)

Design iterative and recursive algorithms to compute $Factorial(n)$. Compare the complexities.

Problem 2 (10 Points)

1. Rank the following functions by order of growth (non-decreasing order)
   $n^2, n!, \ln \ln n, 2^{2^n}, e^n, n^3, n \lg n$
2. Give an example of a single nonnegative function $f(n)$ such that for all functions in part 1, $f(n)$ has no relation.

Problem 3 (10 Points)

Write down the contents of *Any One* of the following arrays after every step of selection sort until the array is completely sorted.

Assume that the arrays given represent their initial arrangement of the numbers. Also compute the number of operations needed. (Comparison and Swapping are the operations)

1.

12	8	-2	23	5	0

2.

31	17	29	11	7	5	3

Problem 4 (10 Points)

What is *Divide and Conquer*?.

Give an example for a *Divide and Conquer* algorithm.

Problem 5 (10 Points)

A Binary Tree is a rooted tree in which every node has at most two children. The root node is said to be in level one. The children of the noded at level n are in level $n + 1$.

Calculate

1. The maximum number of nodes in level h of a binary tree
2. The maximum number of nodes in a binary tree of h levels.
3. How many nodes does a *complete*[1] binary tree with n leaves have?

[1] A binary tree is *complete* if all of its vertices have either zero or two children and all the leaves are at levels l and $l - 1$

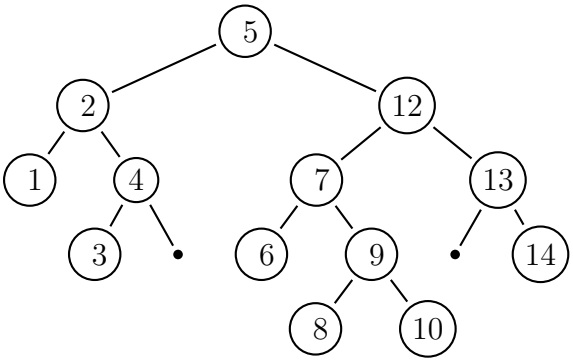
Problem 6 (10 Points)

Review all the sort algorithms taken in the class. Compare their complexities.

Prove that the lower bound for sorting is $n \lg n$

Problem 7

Given is an AVL tree. Perform the operation `insert(11)` on it. Balance the tree.



Problem 8

For an ab-tree of height h (root node is at level zero) and n leaves, prove:

1. $2a^{h-1} \leq n \leq b^h$
2. $\lg_b(n) \leq h \leq \lg_a(\frac{n}{2}) + 1$

Problem 9

Show that the tree defined by the edges traversed in a BFS (starting at v_0) is a shortest paths tree rooted at v_0.