
Fundamental Algorithms

Deadline: November 28, 2007

Problem 1 (10 Points)

What is an undirected tree? Give three definitions of tree and show they are equivalent.

Problem 2 (10 Points)

A Binary Tree is a rooted tree in which every node has at most two children. The root node is said to be in level one. The children of the node at level n are in level $n + 1$.

Calculate

1. The maximum number of nodes in level h of a binary tree
2. The maximum number of nodes in a binary tree of h levels.
3. How many nodes does a *complete*¹ binary tree with n leaves have?

Problem 3 (10 Points)

Analyze the complexity of the `heapify()` function, applied to a complete binary tree containing n nodes. The result should be given as exactly as possible in Landau notation.

Problem 4 (10 Points)

Perform the `heapify()` operation on an array containing the keys $\{0, \dots, 9\}$ where we assume that the keys are stored in the tree in the following order (level-wise, left-to-right):
6; 3, 4; 1, 8, 5, 0; 7, 9, 2 ((levels separated by semicolon))

¹A binary tree is *complete* if all of its vertices have either zero or two children and all the leaves are at levels l and $l - 1$