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Effiziente Algorithmen und Datenstrukturen I

Aufgabe 1

Please prove the following statements:

1. $\forall_{c \in \mathbb{R}_+} c \cdot f(n) = \Theta(f(n))$
2. $f(n) + g(n) = \Omega(f(n))$
3. $g(n) = \mathcal{O}(f(n)) \Rightarrow f(n) + g(n) = \mathcal{O}(f(n))$
4. $\mathcal{O}(f(n)) \cdot \mathcal{O}(g(n)) = \mathcal{O}(f(n) \cdot g(n))$

Aufgabe 2

Prove or disprove the following:

1. $n^3 + 5n^2 - 10 = \mathcal{O}(n^3)$
2. $n \ln n = \mathcal{O}(n)$
3. $n \log n = \Omega(n)$
4. $\log n = o(n)$

Aufgabe 3

Put the following terms in asymptotic order (i.e. $\mathcal{O}(A) \subset \mathcal{O}(B) \subset ...$):

$$n^2 + n \qquad 2^n \qquad \log n \qquad n \log n \qquad 1 \qquad n! \qquad \sqrt{n} \qquad n \qquad n^n$$

Aufgabe 4

Give an asymptotic upper bound (in big-O notation) of time for the worst case for the following code segments (Assume undefined functions like swap, length, etc take $\mathcal{O}(1)$ time):

1.

```
Array A
Array B
if $length(A) < length(B)$ then
    $l := length(A)$
else
    $l := length(B)$
end if
for $i = 1$ to $l$ do
    for $j = l$ downto $i + 1$ do
        if $A[j-1] > A[j]$ then
            swap($A[j-1], A[j]$)
        end if
    end for
    for $k = l$ downto $i + 1$ do
        if $B[k-1] > B[k]$ then
            swap($B[k-1], B[k]$)
        end if
    end for
end for
for $i = 1$ to $length(A)$ do
    if $A[i] \neq B[i]$ then
        output(Sorted list A & B have the first $i - 1$ elements in common.)
    end if
end for
output(Sorted list A & B have the first $i - 1$ elements in common.)
```

2.

```
Function MergeSort(Array A):
if $length(A) = 1$ then
    return A
end if
int mid := $round(length(A)/2)$
Array A1 := $A[1]...A[mid]$
Array A2 := $A[mid + 1]...A[length(A)]$
MergeSort(A1)
MergeSort(A2)
int $a := 1, a1 := 1, a2 := 1$
while $a1 \leq length(A1) \wedge a2 \leq length(A2)$ do
    if $a1 > length(A1) \vee A2[a2] \leq A1[a1]$ then
        $A[a] := A2[a2]$
        $a := a + 1$
        $a2 := a2 + 1$
    else
        $A[a] := A1[a1]$
        $a := a + 1$
        $a1 := a1 + 1$
    end if
end while
```