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Effiziente Algorithmen und Datenstrukturen I

Aufgabe 1 (10 Punkte)

In hashing with chaining, if we modify the chaining scheme so that each list is kept in sorted order, how does it affect the running time for successful searches, unsuccessful searches, insertions, and deletions?

Aufgabe 2 (10 Punkte)

Show that the dynamic set queries $SUCCESSOR$ and $PREDECESSOR$ can be supported in $O(1)$ worst-case time on an augmented order-statistic tree. The asymptotic performance of other operations on order-statistic trees should not be affected.

Aufgabe 3 (10 Punkte)

A forest is an undirected cycle-free graph, i.e. a forest is a graph all of whose connected components are trees. A random graph is obtained by starting with a set of n vertices and adding edges between them at random. In the $G(n,p)$ model, for every pair of vertices $\{a,b\}$, the edge (a,b) occurs independently with probability p. Your goal is to show that for suitable value of p, the probability that G is not a forest is at most a constant (say $\leq \frac{1}{2}$). Note that G not being a forest means that $\exists$ a set $S \subseteq V, |S| = k$, such that the graph induced by S contains at least k edges. Of course, this trivially holds if for example you choose $p = 0$. You should aim for $p = \Omega\left(\frac{1}{n}\right)$. For example, $p = \frac{1}{4e^2 n}$ might be a good choice.

Extra Question:

In the $G(n,m)$ random graph model, we assign equal probability to all graphs with exactly m edges. As in the previous question, but using the $G(n,m)$ model instead of the $G(n,p)$ model, show that the probability that G is not a forest is at most a constant ($\leq \frac{1}{2}$) for $m = \Theta(n)$.

It is voluntary to submit solutions to this extra question, we encourage you to solve it nonetheless.