## Lemma 12 (Chernoff Bounds)

*Let* $X_1, \dots, X_n$ *be* $n$ *independent 0-1 random variables, not necessarily identically distributed. Then for* $X = \sum_{i=1}^{n} X_i$ *and* $\mu = E[X]$, $L \le \mu \le U$, *and* $\delta > 0$

$$\Pr[X \ge (1+\delta)U] < \left(\frac{e^{\delta}}{(1+\delta)^{1+\delta}}\right)^{U} \quad ,$$

*and*

$$\Pr[X \le (1-\delta)L] < \left(\frac{e^{-\delta}}{(1-\delta)^{1-\delta}}\right)^{L} \quad ,$$

## Lemma 13

*For* $0 \leq \delta \leq 1$ *we have that*

$$\left(\frac{e^{\delta}}{(1+\delta)^{1+\delta}}\right)^{U} \leq e^{-U\delta^2/3}$$

*and*

$$\left(\frac{e^{-\delta}}{(1-\delta)^{1-\delta}}\right)^{L} \leq e^{-L\delta^2/2}$$

# Integer Multicommodity Flows

- Given $s_i$-$t_i$ pairs in a graph.
- Connect each pair by a paths such that not too many path use any given edge.

$$\begin{array}{llrcl}
\min & & & & W \\
\textrm{s.t.} & \forall i & \sum_{p\in\mathcal{P}_i} x_p & = & 1 \\
& & \sum_{p:e\in p} x_p & \le & W \\
& & x_p & \in & \{0,1\}
\end{array}$$

# Integer Multicommodity Flows

**Randomized Rounding:**
For each $i$ choose one path from the set $\mathcal{P}_i$ at random according to the probability distribution given by the Linear Programming Solution.

**Theorem 14**

*If* $W^* \geq c \ln n$ *for some constant* $c$*, then with probability at least* $n^{-c/3}$ *the total number of paths using any edge is at most* $W^* + \sqrt{cW^* \ln n}$*.*

# Integer Multicommodity Flows

Let $X_e^i$ be a random variable that indicates whether the path for $s_i$-$t_i$ uses edge $e$.

Then the number of paths using edge $e$ is $Y_e = \sum_i X_e^i$.

$$E[Y_e] = \sum_i \sum_{p \in \mathcal{P}_i : e \in p} x_p^* = \sum_{p : e \in P} x_p^* \leq W^*$$

# Integer Multicommodity Flows

Choose $\delta = \sqrt{(c \ln n)/W^*}$.

Then

$$\Pr[Y_e \geq (1+\delta)W^*] < e^{-W^*\delta^2/3} = \frac{1}{n^{c/3}}$$