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## Efficient Algorithms and Datastructures II

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### Aufgabe 1 (10 Punkte)

Let  $P$  be a given feasible, bounded Linear Program. We know how to find the dual  $D$  of  $P$ . By combining  $P$  and  $D$ , demonstrate a linear program whose only feasible solution corresponds to the feasible solution optimizing the objective value of  $P$  (and similarly for  $D$ ). The new linear program should have constraints linear in the number of constraints of  $P$  and  $D$ . Further, ensure that the new linear program is infeasible if either  $P$  or  $D$  is infeasible.

### Aufgabe 2 (10 Punkte)

Using the above idea demonstrate that we can reduce (in polynomial time,) the problem of solving an LP to that of finding whether an LP is feasible. In other words show that finding whether an LP is feasible is as tough (in terms of complexity) as solving an LP for optimality.

### Aufgabe 3 (10 Punkte)

We wish to find an  $x \in \mathbb{R}$  having a certain property  $P$  or to prove that no  $x \in \mathbb{R}$  has property  $P$ . We know that absolute value of any  $x$  with property  $P$  is bounded by  $b$  and that the set  $S$  of real numbers having property  $P$  is either empty or an interval of length at least  $\ell$ . Given  $x \in \mathbb{R}$ , we can test whether  $x$  has property  $P$ . If  $x$  does not have property  $P$ , we can tell whether  $S \in [-\infty, x]$  or  $S \in [x, \infty]$ . Give an algorithm to solve this problem efficiently. What is the number of steps taken by your algorithm?