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Spring Semester 2013
Problem Set 4
May 14, 2013

Complexity Theory

Due date: May 28, 2013 **before** *class!*

Problem 1 (10 Points)

(i) Assume $A \preceq_m^p B$. Show that then also $\overline{A} \preceq_m^p \overline{B}$.

(ii) Show that if a complexity class $\mathcal{C}$ is closed under $\preceq_m^p$, then so is co$\mathcal{C}$.

(iii) Show that co$\mathcal{NP}$ is closed under union and intersection.

Problem 2 (10 Points)

(i) Show that $\mathcal{NP}$ and co$\mathcal{NP}$ both are subsets of the set of languages which are polynomial-time Turing-reducible to Sat.

(ii) Prove that if $\mathcal{NP}$ was equal to the set of languages which are polynomial-time Turing-reducible to Sat, it would follow that $\mathcal{NP} = \text{co}\mathcal{NP}$.

Problem 3 (10 Points)

Recall the construction of a graph G for showing that HamPath is $\mathcal{NP}$-complete (see the textbook for further references). Elaborate the details of the proof by showing the claim

$$G \in \text{HamPath} \implies \varphi \in \text{Sat}.$$

(It is done in the textbook; however, the last part misses the details. What you should elaborate is being stated in the textbook as 'It is not hard to see that [...]'.)

Problem 4 (10 Points)

Define the following two covering problems:

- A *vertex cover* of a graph $G = (V, E)$ is a set of vertices $V' \subseteq V$, where every edge in E is incident to at least one vertex in V'.
  Let Vertex Cover $= \{(G, k) : G$ has a vertex cover of size at most $k\}$.
- Given a set U, and a family S of subsets of U, a *set cover* of U is a subfamily of sets $C \subseteq S$ whose union is U.
  Let Set Cover $= \{(U, S, k) : U$ has a set cover of size at most $k\}$.

Show the following two claims.

(i) Vertex Cover is $\mathcal{NP}$-complete.

(ii) Set Cover is $\mathcal{NP}$-complete.

Problem 5 (10 Points)

Define a *regular expression* over $\{0, 1\}$ as

$$r ::= 0 \mid 1 \mid rr \mid r|r.$$

The problem RegExpEq is about the question whether two languages defined by two different regular expressions are identical. A special case of this is the language RegExpEq_*, which contains all regular expressions that create the language of all words Σ^*.
Show that RegExpEq_* is co$\mathcal{NP}$-complete.

Problem 6 (10 Points)

Rewrite the proof of the Time Hierarchy Theorem, but use the whole string x^* as a description of a TM, not just a prefix of x^*.