
Complexity Theory

Due date: June 25, 2013 before class!

Problem 1 (10 Points)

Given $n = 2^k$ and two n -bit numbers, the task is to add these numbers with a parallel algorithm.

- (i) Describe an approach on how to compute the behavior of the i th carry bit in relation to the $(i - 1)$ st carry bit.
- (ii) Describe how to compute this for all n carry bits in only $O(\log n)$ bit steps.

Problem 2 (10 Points)

Using Problem 1, describe a parallel algorithm for adding two n -bit numbers in $O(\log n)$ steps.

Problem 3 (10 Points)

Prove the Non-uniform Hierachy Theorem:

For functions $T, T' : \mathbb{N} \rightarrow \mathbb{N}$ with $n < T(n) < T'(n) < \frac{2^n}{100n}$ and $T \log T = o(T')$, it follows that $\mathbf{SIZE}(T(n)) \subsetneq \mathbf{SIZE}(T'(n))$.

Hint: The proof idea for a linear and a quadratic function is given in the textbook.

Problem 4 (10 Points)

Show that $\mathbf{NL} \subseteq \mathbf{NC}$.