Technische Universität München
Fakultät für Informatik
Lehrstuhl für Effiziente Algorithmen
Prof. Dr. Harald Räcke
Chintan Shah

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# Efficient Algorithms and Datastructures I

## Question 1 (10 Points)

The mean $M$ of a set of $k$ integers $\{x_1, x_2, \ldots, x_k\}$ is defined as

$$M = \frac{1}{k}\sum_{i=1}^{k} x_i.$$

We want to maintain a data structure $D$ on a set of integers under the normal INIT, INSERT, DELETE, and FIND operations, as well as a MEAN operation, defined as follows:

1. INIT($D$): Create an empty structure $D$.
2. INSERT($D, x$): Insert $x$ in $D$.
3. DELETE($D, x$): Delete $x$ from $D$.
4. FIND($D, x$): Return pointer to $x$ in $D$.
5. MEAN($D, a, b$): Return the mean of the set consisting of elements $x$ in $D$ with $a \le x \le b$.

Describe how to modify a standard red-black tree in order to implement $D$, such that INIT is supported in $O(1)$ time and INSERT, DELETE, FIND, and MEAN are supported in $O(\log n)$ time.

## Question 2 (10 Points)

In double hashing, if we use the hash function $h(k,i) = (h_1(k) + ih_2(k)) \bmod m$, show that when $m$ and $h_2(k)$ have greatest common divisor $d \ge 1$ for some key $k$, then an unsuccessful search for key $k$ examines $\frac{1}{d}$th of the hash table before returning to slot $h_1(k)$.
(*Note:* When $d = 1$, i.e. when $m$ and $h_2(k)$ are relatively prime, the search may examine the entire hash table.)

## Question 3 (10 Points)

Let $U = \{0, \ldots, p-1\}$ for a prime $p$. For $x \in \mathbb{Z}_p$, define the hash function $h_{a,b}(x)$ as

$$h_{a,b}(x) = (ax + b \mod p) \mod n$$

Consider the class of hash functions

$$\mathcal{H} = \{h_{a,b} | a, b \in \mathbb{Z}_p\}$$

(a) Show that $\mathcal{H}$ is not universal.

(b) Show that $\mathcal{H}$ is $(1.1, 2)$ independent for $p$ sufficiently large.

(c) Why would you not choose $\mathcal{H}$ as a class of hash functions?