WS 2014/15

Automaten und Formale Sprachen
Automata and Formal Languages

Ernst W. Mayr

Fakultät für Informatik
TU München

http://www14.in.tum.de/lehre/2014WS/afs/

Wintersemester 2014/15

Chapter 0 Organizational Matters

- Lectures:
  - 4SWS Tue 08:30–10:00 (MI 00.13.009A)
    Fri 10:15–11:45 (MI 00.13.009A)
    Compulsory elective in area Theoretical Computer Science
    Module no. IN2041
- Exercises/Tutorial:
  - 2SWS Tutorial: Tue 12:00–13:30 (03.11.018)
  - Tutor: Moritz Fuchs
- Valuation:
  - 4V+2ZÜ, 8 ECTS points
- Office hours:
  - Fri 12:00–13:00 and by appointment

- Tutor sessions:
  - Moritz Fuchs, MI 03.09.037 (fuchsmo@in.tum.de)
    Office hours: Tue 14:00–16:00
- Secretariat:
  - Mrs. Lissner, MI 03.09.052 (lissner@in.tum.de)

- Problem sets and final exam:
  - problem sets are made available on Tuesdays on the course webpage
  - must be turned in a week later before class, if you want them marked
  - are discussed in the tutor session
  - probably 12 problem sets
- Exam:
  - final exam: Wednesday, February 11, 2015, 11:30–14:30, room MI HS3
  - the final exam is closed book, no auxiliary means are permitted except for one sheet of DIN-A4 paper, handwritten by yourself
  - to pass the final exam, it is necessary to obtain at least 40% of the point total

- Prerequisites:
  - Fundamentals of Algorithms and Data Structures (GAD)
  - Introduction to Theory of Computer Science (THEO)
- Supplementary courses:
  - Logics
  - Model Checking
  - Verification
  - ...
- Webpage:

  http://www14.in.tum.de/lehre/2014WS/afs/

1. Planned topics for the course

- Automata on finite words
  - Automata classes and conversions
    - Regular expressions, deterministic and nondetermistic automata
    - Conversion algorithms
  - Minimization and reduction
    - Minimizing DFAs
    - Reducing NFAs
  - Boolean operations and tests
    - Implementation on DFAs
    - Membership, complement, union, intersection, emptiness, universality, inclusion
    - Implementation on NFAs
  - Operations on relations
    - Projection, join, post, pre
  - Operations on finite universes: decision diagrams
  - Automata and logic
  - Applications: pattern-matching, verification, Presburger arithmetic

- Automata on infinite words
  - Automata classes and conversions
    - Omega-regular expressions
    - Büchi, Streett, Rabin, and Muller automata
  - Boolean operations
    - Union and intersection
    - Complement
  - Checking emptiness
  - Applications: verification using temporal logic

2. Literature

John E. Hopcroft, Rajeev Motwani, Jeffrey D. Ullman:
Introduction to Automata Theory, Languages and Computation,
Addison-Wesley Longman, 3rd edition, 2006

John Martin:
Introduction to Languages and the Theory of Computation,
McGraw-Hill, 2002

Michael Sipser:
Introduction to the Theory of Computation,
International Edition, Thomson Course Technology:
Australia-Canada-Mexico-Singapore-Spain-United Kingdom-United States, 2006

Erich Grädel, Wolfgang Thomas, Thomas Wilke (eds.):
Automata, logics, and infinite games: a guide to current research,
LNCS **2500**, Springer-Verlag, 2002

Dominique Perrin, Jean-Eric Pin:
Infinite Words: Automata, Semigroups, Logic and Games,
Academic Press, 2004

Also see Javier Esparza's lecture notes from WS2012/13, onto which this incarnation of the course is also based (but which contain much more material).

Further relevant research papers will be made available during the course.

3. Notational conventions

We use standard notation and basic concepts, as detailed e.g., in the introductory course on

Discrete Structures, IN0015

http://wwwmayr.in.tum.de/lehre/2012WS/ds/index.html.en

4. Mathematical and Notational Basics

4.1 Sets

Example 1

$$A_1 = \{2, 4, 6, 8\};$$
$$A_2 = \{0, 2, 4, 6, \dots\} = \{n \in \mathbb{N}_0; n \text{ even}\}$$

Notation:

$x \in A \Leftrightarrow A \ni x$	x element of A
$x \notin A$	x not element of A
$B \subseteq A$	B subset of A
$B \subsetneq A$	B proper subset of A
$\emptyset$	empty set, as opposed to:
$\{\emptyset\}$	set with empty set as (only) element

Special Sets:

- $\mathbb{N} = \{1, 2, \ldots\}$
- $\mathbb{N}_0 = \{0, 1, 2, \ldots\}$
- $\mathbb{Z}$ = set of the integers
- $\mathbb{Q}$ = set of the rational numbers
- $\mathbb{R}$ = set of the real numbers
- $\mathbb{C}$ = set of the complex numbers
- $\mathbb{Z}_n = \{0, 1, \ldots, n-1\}$ residue classes for division by n
- $[n] = \{1, 2, \ldots, n\}$

Operations on Sets:

- $|A|$ cardinality of the set A
- $A \cup B$ set union
- $A \cap B$ set intersection
- $A \setminus B$ set difference
- $A \vartriangle B := (A \setminus B) \cup (B \setminus A)$ symmetric difference
- $A \times B := \{(a, b); a \in A, b \in B\}$ cartesian product
- $A \uplus B$ disjoint union; the elements are distinguished according to their origin
- $\bigcup_{i=0}^{n} A_i$ union of the sets $A_0, A_1, \ldots, A_n$
- $\bigcap_{i \in I} A_i$ intersection of the sets A_i mit $i \in I$
- $\mathsf{P}(M) := 2^M := \{N; N \subseteq M\}$ power set of the set M

Example 2

Für $M = \{a, b, c, d\}$ ist

$$
\begin{aligned}
P(M) \quad = \quad \{ \quad & \emptyset, \{a\}, \{b\}, \{c\}, \{d\}, \\
& \{a, b\}, \{a, c\}, \{a, d\}, \{b, c\}, \{b, d\}, \{c, d\}, \\
& \{a, b, c\}, \{a, b, d\}, \{a, c, d\}, \{b, c, d\}, \\
& \{a, b, c, d\} \\
\}
\end{aligned}
$$

Theorem 3

Let the cardinality of set M be n, $n \in \mathbb{N}$. Then $P(M)$ has 2^n elements!

Proof.

Let $M = \{a_1, \ldots, a_n\}$, $n \in \mathbb{N}$. To obtain a set $L \in P(M)$ (*i.e.* $L \subseteq M$), we have, for each $i \in [n]$, the (independent) choice to add a_i to L or not. This results in $2^{|[n]|} = 2^n$ different possibilities for L. □

Remarks:

1. The above theorem also holds for $n = 0$, *i.e.*, the empty set $M = \emptyset$.
2. The empty set is a subset of every set.
3. $P(\emptyset)$ has exactly $\emptyset$ as element.

4.2 Relations and Mappings

Let $A_1, A_2, \ldots, A_n$ be sets. A relation R over $A_1, \ldots, A_n$ is a subset

$$R \subseteq A_1 \times A_2 \times \ldots \times A_n = \bigtimes_{i=1}^{n} A_i$$

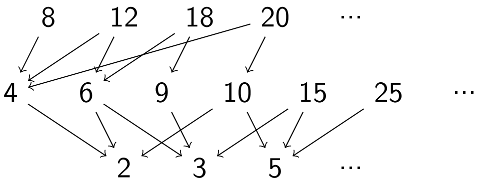
Other notation (infix notation) for $(a, b) \in R$: aRb.

Properties of relations ($R \subseteq A \times A$):

- reflexive: $(a, a) \in R \quad \forall a \in A$
- symmetric: $(a, b) \in R \Rightarrow (b, a) \in R \quad \forall a, b \in A$
- asymmetric: $(a, b) \in R \Rightarrow (b, a) \notin R \quad \forall a, b \in A$
- antisymmetric: $\big[(a, b) \in R \land (b, a) \in R\big] \Rightarrow a = b \quad \forall a, b \in A$
- transitive: $\big[(a, b) \in R \land (b, c) \in R\big] \Rightarrow (a, c) \in R \quad \forall a, b, c \in A$
- equivalence relation: reflexiv, symmetrisch und transitiv
- partial order (aka *partially ordered set*, *poset*): reflexive, antisymmetric and transitive

Example 4

Let $(a, b) \in R$ iff $a|b$, *i.e.*, "a divides b", $a, b \in \mathbb{N} \setminus \{1\}$.
The graphical representation of R without reflexive and transitive arcs is called Hasse diagram:



In the diagram, $a|b$ is denoted by an arc $b \longrightarrow a$.
The relation $|$ is a *partial order*.