Muller automata

- A nondeterministic Muller automaton (NMA) has a collection $\{F_0, F_1, \dots, F_{m-1}\}$ of sets of accepting states.
- A run is accepting if the set of states it visits infinitely often is equal to one of the sets in the collection.

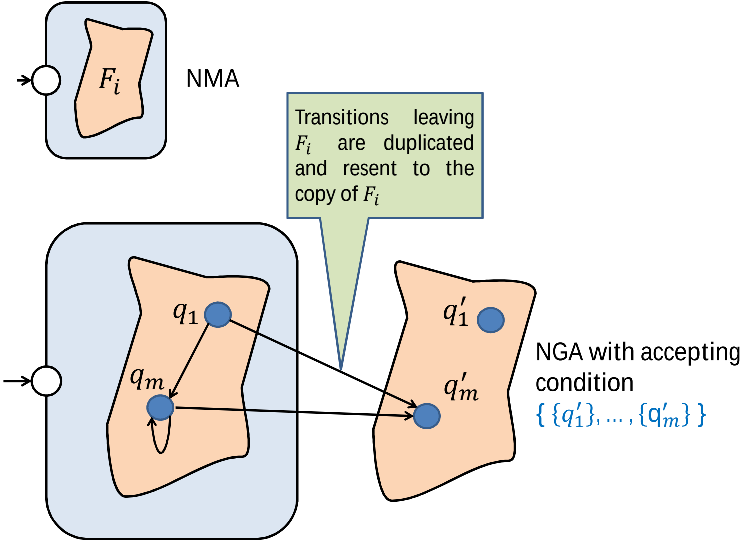
From Büchi to Muller automata

- Let A be a NBA with set F of accepting states.
- A set of states of A is good if it contains some state of F.
- Let G be the set of all good sets of A.
- Let A' be "the same automaton" as A, but with Muller condition G.
- Let ρ be an arbitrary run of A and A'. We have

	ρ is accepting in A
iff	$\inf(\rho)$ contains some state of F
iff	$\inf(\rho)$ is a good set of A
iff	ρ is accepting in A'

From Muller to Büchi automata

- Let A be a NMA with condition $\{F_0, F_1, \ldots, F_{m-1}\}$.
- Let $A_0, \ldots, A_{m-1}$ be NMAs with the same structure as A but Muller conditions $\{F_0\}, \{F_1\}, \ldots, \{F_{m-1}\}$ respectively.
- We have: $L(A) = L(A_0) \cup \ldots \cup L(A_{m-1})$
- We proceed in two steps:
  1. we construct for each NMA A_i an NGA $A_i{}'$ such that $L(A_i) = L(A_i')$
  2. we construct an NGA A' such that
     $$L(A') = L(A'_0) \cup \ldots \cup L(A'_{m-1})$$


F_i
NMA
Transitions leaving F_i are duplicated and resent to the copy of F_i
q_1
q_m
q'_1
q'_m
NGA with accepting condition
$\{ \{q'_1\}, \dots, \{q'_m\} \}$

NMA1toNGA(A)

Input: NMA $A = (Q, \Sigma, q_0, \delta, \{F\}$

Output: NGA $A = (Q', \Sigma, q'_0, \delta', \mathcal{F}')$

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Q', δ', F' ← ∅
q'_0 ← [q_0, 0]
W ← {[q_0, 0]}
while W ≠ ∅ do
    pick [q, i] from W; add [q, i] to Q'
    if q ∈ F and i = 1 then add {[q, 1]} to F'
    for all a ∈ Σ, q' ∈ δ(q, a) do
        if i = 0 then
            add ([q, 0], a, [q', 0]) to δ'
            if [q', 0] ∉ Q' then add [q', 0] to W
            if q' ∈ F then
                add ([q, 0], a, [q', 1]) to δ'
                if [q', 1] ∉ Q' then add [q', 1] to W
        else /* i = 1 */
            if q' ∈ F then
                add ([q, 1], a, [q', 1]) to δ'
                if [q', 1] ∉ Q' then add [q', 1] to W
return (Q', Σ, q'_0, δ', F')
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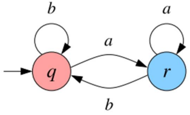
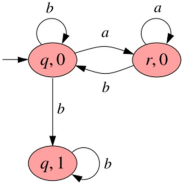


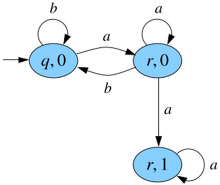
$\mathcal{F} = \{F_0, F_1\}$

$F_0 = \{q\}$

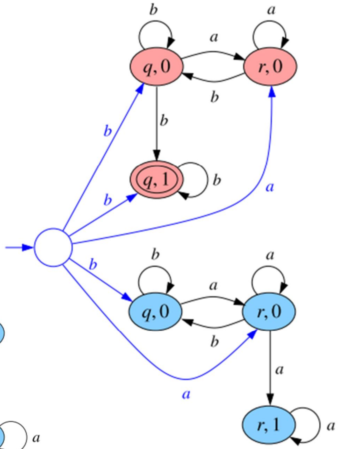
$F_1 = \{r\}$



$\mathcal{F}'_0 = \{\ [q, 1]\ \}$



$\mathcal{F}'_1 = \{\ [r, 1]\ \}$

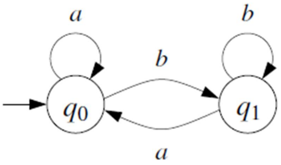


Equivalence of NMAs and DMAs

- Theorem (Safra): Any NBA with n states can be effectively transformed into a DMA of size $n^{O(n)}$.

  Proof: Omitted.

- DMA for $(a + b)^* b^\omega$:



with accepting condition $\{\{q_1\}\}$

- Question: Are there other classes of omega-automata with
  - the same expressive power as NBAs or NGAs, and
  - with equivalent deterministic and nondeterministic versions?
- Answer: Yes, Muller automata

Is the quest over?

- Recall the translation NBA $\Rightarrow$ NMA
- The NMA has the same structure as the NBA; its accepting condition are all the good sets of states.
- The translation has exponential complexity.

New question: Is there a class of ω-automata with
- the same expressive power as NBAs,
- equivalent deterministic and nondeterministic versions, and
- polynomial conversions to and from Büchi automata?

Rabin automata

- The acceptance condition is a set of pairs $\{ \langle F_0, G_0 \rangle, \ldots, \langle F_{m-1}, G_{m-1} \rangle \}$
- A run ρ is accepting if there is a pair $\langle F_i, G_i \rangle$ such that ρ visits the set F_i infinitely often and the set G_i finitely often.
- Translations NBA ⇒ NRA and NRA ⇒ NBA are left as an exercise.
- Theorem (Safra): Any NBA with n states can be effectively transformed into a DRA with $n^{O(n)}$ states and $O(n)$ accepting pairs.