
Automata and Formal Languages

Due November 11, 2014 before class!

Exercise 1 (10 points)

Prove that $\binom{n}{n/2} \in \Theta\left(\frac{2^n}{\sqrt{n}}\right)$.

Exercise 2 (Subsumption - 10 points)

In the lecture we saw the subsumption heuristic for deciding whether a given NFA is universal or not.

- (a) Give an example where non-minimal states are added to the worklist.
- (b) Which data structure would you use in practice for the worklist? Justify your decision.

Exercise 3 (Scattered subword - 10 points)

We say that $u = a_1 \dots a_n$ is a *scattered subword* of w (short: $u \triangleleft w$) if $w = w_0 a_1 w_1 a_2 w_2 \dots w_{n-1} a_n w_n$ with $w_0, \dots, w_n \in \Sigma^*$. Let L be a regular language. Show:

- (a) $L' = \{u \in \Sigma^* \mid \exists w \in L : w \triangleleft u\}$ is regular.
- (b) $L'' = \{u \in \Sigma^* \mid \exists w \in L : u \triangleleft w\}$ is regular.

Exercise 4 (Halves - 10 points)

Let L be a regular language and define $L_{\frac{1}{2}} = \{x \mid \exists y : |x| = |y| \wedge xy \in L\}$.
Prove: $L_{\frac{1}{2}}$ is regular.