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- ▶ An exact analysis (e.g. *exactly* counting the number of operations in a RAM) may be hard, but wouldn't lead to more precise results as the computational model is already quite a distance from reality.
- ▶ A linear speed-up (i.e., by a constant factor) is always possible by e.g. implementing the algorithm on a faster machine.
- ▶ Running time should be expressed by simple functions.

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Asymptotic Notation

Formal Definition

Let f denote functions from \mathbb{N} to \mathbb{R}^+ .

- ▶ $\mathcal{O}(f) = \{g \mid \exists c > 0 \exists n_0 \in \mathbb{N}_0 \forall n \geq n_0 : [g(n) \leq c \cdot f(n)]\}$
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There is an equivalent definition using limes notation (**assuming that the respective limes exists**). f and g are functions from \mathbb{N}_0 to \mathbb{R}_0^+ .

$$\blacktriangleright g \in \mathcal{O}(f): 0 \leq \lim_{n \rightarrow \infty} \frac{g(n)}{f(n)} < \infty$$

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Abuse of notation

1. People write $f = \mathcal{O}(g)$, when they mean $f \in \mathcal{O}(g)$. This is **not** an equality (how could a function be equal to a set of functions).
2. People write $f(n) = \mathcal{O}(g(n))$, when they mean $f \in \mathcal{O}(g)$, with $f: \mathbb{N} \rightarrow \mathbb{R}^+, n \mapsto f(n)$, and $g: \mathbb{N} \rightarrow \mathbb{R}^+, n \mapsto g(n)$.
3. People write e.g. $h(n) = f(n) + o(g(n))$ when they mean that there exists a function $z: \mathbb{N} \rightarrow \mathbb{R}^+, n \mapsto z(n), z \in o(g)$ such that $h(n) = f(n) + z(n)$.
4. People write $\mathcal{O}(f(n)) = \mathcal{O}(g(n))$, when they mean $\mathcal{O}(f(n)) \subseteq \mathcal{O}(g(n))$. Again this is not an equality.

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Asymptotic Notation in Equations

How do we interpret an expression like:

$$2n^2 + 3n + 1 = 2n^2 + \Theta(n)$$

Here, $\Theta(n)$ stands for an **anonymous function** in the set $\Theta(n)$ that makes the expression true.

Note that $\Theta(n)$ is on the right hand side, otw. this interpretation is wrong.

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How do we interpret an expression like:

$$2n^2 + \mathcal{O}(n) = \Theta(n^2)$$

Regardless of how we choose the anonymous function $f(n) \in \mathcal{O}(n)$ there is an anonymous function $g(n) \in \Theta(n^2)$ that makes the expression true.

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How do we interpret an expression like:

$$\sum_{i=1}^n \Theta(i) = \Theta(n^2)$$

Careful!

“It is understood” that every occurrence of an Θ -symbol (or $\Theta, \Omega, o, \omega$) on the left represents *one anonymous function*.

Hence, the left side is **not** equal to

$$\Theta(1) + \Theta(2) + \dots + \Theta(n-1) + \Theta(n)$$

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Asymptotic Notation in Equations

We can view an expression containing asymptotic notation as generating a set:

$$n^2 \cdot \mathcal{O}(n) + \mathcal{O}(\log n)$$

represents

$$\left\{ f : \mathbb{N} \rightarrow \mathbb{R}^+ \mid f(n) = n^2 \cdot g(n) + h(n) \right. \\ \left. \text{with } g(n) \in \mathcal{O}(n) \text{ and } h(n) \in \mathcal{O}(\log n) \right\}$$

Asymptotic Notation in Equations

Then an asymptotic equation can be interpreted as containment btw. two sets:

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represents

$$n^2 \cdot \mathcal{O}(n) + \mathcal{O}(\log n) \subseteq \Theta(n^2)$$

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Lemma 1

Let f, g be functions with the property

$\exists n_0 > 0 \forall n \geq n_0 : f(n) > 0$ (the same for g). Then

- ▶ $c \cdot f(n) \in \Theta(f(n))$ for any constant c
- ▶ $\mathcal{O}(f(n)) + \mathcal{O}(g(n)) = \mathcal{O}(f(n) + g(n))$
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- ▶ $\mathcal{O}(f(n)) + \mathcal{O}(g(n)) = \mathcal{O}(\max\{f(n), g(n)\})$

The expressions also hold for Ω . Note that this means that $f(n) + g(n) \in \Theta(\max\{f(n), g(n)\})$.

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Comments

- ▶ Do not use asymptotic notation within induction proofs.
- ▶ For any constants a, b we have $\log_a n = \Theta(\log_b n)$.
Therefore, we will usually ignore the base of a logarithm within asymptotic notation.
- ▶ In general $\log n = \log_2 n$, i.e., we use 2 as the default base for the logarithm.

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In general asymptotic classification of running times is a good measure for comparing algorithms:

- ▶ If the running time analysis is tight and actually occurs in practise (i.e., the asymptotic bound is not a purely theoretical worst-case bound), then the algorithm that has better asymptotic running time will always outperform a weaker algorithm for large enough values of n .
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 - ▶ Algorithm A. Running time $f(n) = 1000 \log n = \mathcal{O}(\log n)$.
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